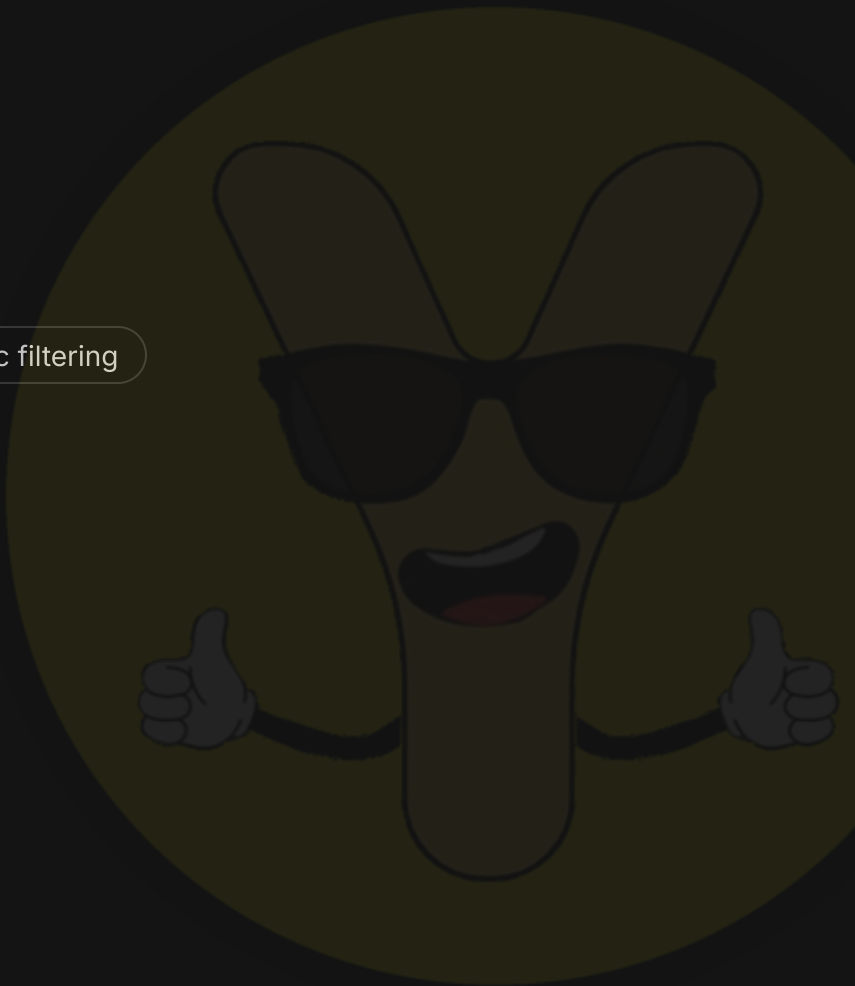
AKTU · 3RD YEAR · SEMESTER 5 · BCS057

Image Processing

Unit 2 — Image Enhancement

- Spatial Domain: Gray level transformations
- Histogram processing
- Basics of Spatial Filtering
- Smoothing and Sharpening Spatial Filtering
- Frequency Domain: Introduction to Fourier Transform
- Smoothing and Sharpening frequency domain filters
- Ideal, Butterworth and Gaussian filters
- Homomorphic filtering
- Color image enhancement



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Unit 2 Overview: Image Enhancement

Image enhancement improves visual quality or transforms images into forms better suited for human interpretation and machine analysis.

- Spatial Domain methods operate directly on image pixels: $g(x, y) = T[f(x, y)]$.
- Frequency Domain methods modify Fourier transform coefficients of an image: $G(u, v) = H(u, v)F(u, v)$.
- Point processing techniques map single intensity values using non-linear and linear transformations.
- Histogram processing adjusts global contrast by redistributing dynamic gray-level ranges.
- Spatial filtering uses spatial masks for noise reduction (smoothing) and edge detection (sharpening).
- Frequency filtering provides global control over spatial frequency spectrum components.

Gray Level Intensity Transformations

POINT PROCESSING TRANSFORMATION

A spatial domain technique where the output intensity s at pixel (x, y) depends strictly on the input intensity r at that coordinate via a mapping function $s = T(r)$.

- Linear Transformations: Include Image Negative $s = (L - 1) - r$ to highlight dark details, and Identity mapping.
- Logarithmic Transformations: Expressed as $s = c \cdot \log(1 + r)$; expands dark pixels while compressing bright values.
- Power-Law (Gamma) Transformations: Expressed as $s = c \cdot r^{\gamma}$; used for monitor display calibration and dynamic range adjustment.

Power-Law & Piecewise-Linear Transformations

Non-linear gray level mappings adjust contrast across specific target intensity ranges.

- Power-Law Equation: $s = c \cdot r^{\gamma}$, where $\gamma < 1$ expands dark regions and $\gamma > 1$ expands bright regions.
- Contrast Stretching
  - Expands low-contrast images across full dynamic range using piecewise-linear mapping functions.
  - Stretches key gray-level intervals while preserving surrounding intensity boundaries.
- Intensity-Level Slicing: Highlights a specific range of gray levels while dampening or preserving other background levels.
- Bit-Plane Slicing: Separates pixel intensity into individual binary bits to isolate high-order detail components.

Image Histogram & Probability Density Function

INTENSITY HISTOGRAM

A discrete function $h(r_k) = n_k$, where r_k is the k-th intensity value in range $[0, L-1]$ and n_k is the number of pixels having intensity r_k.

- Normalized Histogram: Computed as $p(r_k) = \frac{n_k}{MN}$, giving empirical probability density of level r_k.
- Dark Image: Histogram components concentrated near low (dark) intensity values.
- Bright Image: Histogram components biased toward high (bright) intensity values.
- Low-Contrast Image: Narrow histogram concentrated near middle of intensity scale.
- High-Contrast Image: Broad histogram spanning almost entire intensity range uniformly.

Histogram Equalization Algorithm

Automatically redistributes gray levels uniformly across dynamic range to maximize global image contrast.

- Transformation Function: $s = T(r) = (L-1)\int_0^r p_r(w)dw$, yielding uniform probability density function $p_s(s)$.
- Discrete Form: $s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{L-1}{MN}\sum_{j=0}^{k} n_j$.
- Rounding Step: Calculated mapped value s_k is rounded to nearest integer index in range $[0, L-1]$.
- Monotonicity: Function $T(r)$ is strictly monotonic to preserve original relative brightness ordering.
- Adaptive Histogram Equalization (AHE): Computes local histograms for localized contrast enhancement.

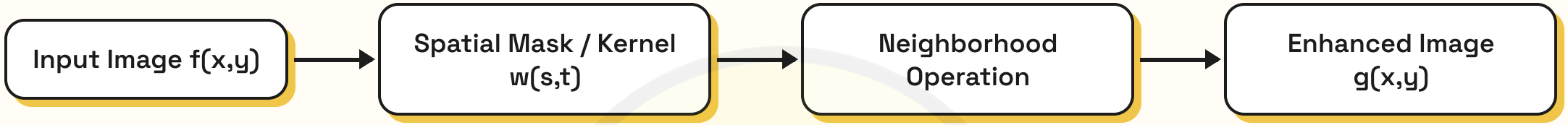
Numerical Example: Histogram Equalization

Given a 3-bit image (L=8) of size 64 × 64 = 4096 pixels with intensity counts: n_0=790, n_1=1023, n_2=850, n_3=656, n_4=329, n_5=245, n_6=122, n_7=81. Perform histogram equalization.

1. Step 1: Compute PDF p(r_k) = n_k / 4096. p = [0.19, 0.25, 0.21, 0.16, 0.08, 0.06, 0.03, 0.02].
2. Step 2: Compute CDF C(r_k) = sum(p(r_j)). C = [0.19, 0.44, 0.65, 0.81, 0.89, 0.95, 0.98, 1.00].
3. Step 3: Multiply CDF by (L-1)=7. Values = [1.33, 3.08, 4.55, 5.67, 6.23, 6.65, 6.86, 7.00].
4. Step 4: Round to nearest integer. s_0=1, s_1=3, s_2=5, s_3=6, s_4=6, s_5=7, s_6=7, s_7=7.

Mapped Gray Levels: r_0→1, r_1→3, r_2→5, r_3→6, r_4→6, r_5→7, r_6→7, r_7→7.

Spatial Filtering Pipeline



Mechanics of spatial filtering using a neighborhood mask sliding over an image.

Correlation vs Convolution Mechanics

Parameter	Spatial Correlation	Spatial Convolution
Definition	Sliding filter mask over image and computing sum of products	Flipping filter mask by 180 degrees before sliding and computing product sum
Formula	$g(x,y) = \sum\sum w(s,t) f(x+s, y+t)$	$g(x,y) = \sum\sum w(s,t) f(x-s, y-t)$
Symmetric Mask Effect	Identical result to convolution when filter mask is symmetric	Identical result to correlation when filter mask is symmetric
Primary Purpose	Feature matching and pattern template identification	Linear filtering, smoothing, and derivative computation

Smoothing Spatial Filters (Lowpass)

Reduces noise and removes high-frequency details prior to structural feature extraction.

- Linear Box Filter: Standard averaging filter mask with equal weights $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$.
- Weighted Average Filter: Center pixel multiplied by higher weight $\frac{1}{16}\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}$ to reduce blur.
- Non-Linear Median Filter: Replaces center pixel value with median of neighborhood pixels.
- Impulse Noise Removal: Median filter excels at removing salt-and-pepper noise without blurring sharp edges.
- Order-Statistic Max/Min Filters: Replaces center pixel with maximum or minimum value in neighborhood.

Sharpening Spatial Filters (Highpass)

Highlights fine intensity transitions, edges, and structural discontinuities in an image.

- First-Order Derivative: Gradient magnitude $\nabla f = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$, where $|\nabla f| \approx |g_x| + |g_y|$.
- Sobel Operators: Gradient masks with smoothing effect: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$, $M_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$.
- Prewitt Operators: Simpler gradient masks: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$, $M_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.
- Second-Order Derivative: Laplacian operator $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. Isotropic mask: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Laplacian Sharpening & Unsharp Masking

Combining derivative edge masks with original image restores sharp detail while preserving background content.

- Laplacian Enhancement: $g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$, where $c = -1$ if center mask coefficient is negative.
- Step 1 (Unsharp Masking): Blur original image: $\bar{f}(x,y) = \text{Smooth}\{f(x,y)\}$.
- Step 2 (Mask Generation): Subtract blurred image from original: $g_{\text{mask}}(x,y) = f(x,y) - \bar{f}(x,y)$.
- Step 3 (Weighted Addition): Add weighted mask to original: $g(x,y) = f(x,y) + k \cdot g_{\text{mask}}(x,y)$.
- Highboost Filtering: Achieved when weighting factor $k > 1$, significantly boosting high-frequency detail.

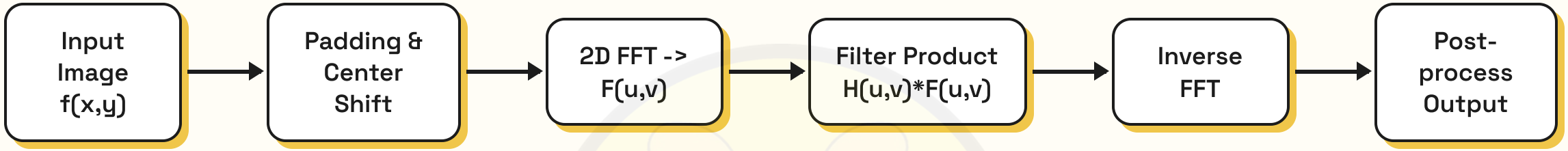
2D Discrete Fourier Transform (DFT)

2D DISCRETE FOURIER TRANSFORM

Converts a spatial domain image $f(x, y)$ of size $M \times N$ into its frequency spectrum representation $F(u, v)$.

- Forward 2D DFT: $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})}$.
- Inverse 2D DFT: $f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(\frac{ux}{M} + \frac{vy}{N})}$.
- Magnitude Spectrum: $|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$, representing spatial rates of intensity changes.
- DC Component: $F(0,0) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$, proportional to total average image intensity.

Frequency Domain Filtering Steps



Sequential processing pipeline for image enhancement in frequency domain.

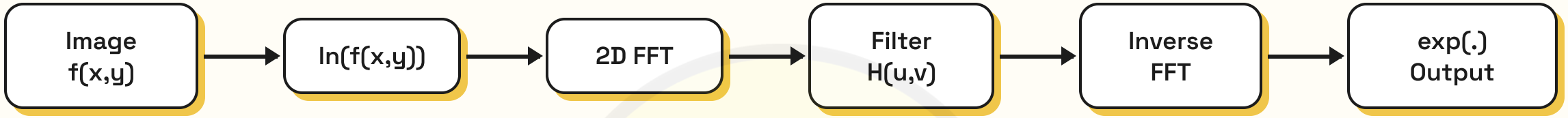
Lowpass Frequency Domain Filters (LPF)

Filter Type	Transfer Function H(u,v)	Characteristics & Artifacts
Ideal Lowpass (ILPF)	$H(u,v) = 1 \text{ if } D(u,v) \leq D_0 \text{ else } 0$	Sharp cutoff; causes severe ringing artifacts (Gibbs phenomenon) in spatial domain.
Butterworth LPF (BLPF)	$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$	Smooth transition controlled by order n; clear trade-off between smoothing and ringing.
Gaussian LPF (GLPF)	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$	Completely smooth transition; guarantees zero ringing in both spatial and frequency domains.

Highpass Frequency Domain Filters (HPF)

Filter Type	Transfer Function H(u,v)	Properties
Ideal Highpass (IHPF)	$H(u,v) = 0$ if $D(u,v) \leq D_0$ else 1	Attenuates low frequencies completely; produces strong ringing artifacts in spatial output.
Butterworth HPF (BHPF)	$H(u,v) = \frac{1}{1+[D_0/D(u,v)]^{2n}}$	Smooth highpass transition; filter order n controls steepness near cutoff frequency D_0.
Gaussian HPF (GHPF)	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$	Smooth attenuation of low frequencies with zero ringing; provides clean edge enhancement.

Homomorphic Filtering Pipeline



Separation of illumination and reflectance components using logarithmic frequency transformation.

Homomorphic Filtering Mechanics

Separates non-uniform illumination from scene reflectance to simultaneously compress dynamic range and increase contrast.

- Image Model: $f(x,y) = i(x,y) \cdot r(x,y)$, where i is illumination (low frequency) and r is reflectance (high frequency).
- Logarithmic Separation: $Z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$.
- Fourier Domain Transformation: $\mathcal{F}\{Z(x,y)\} = F_i(u,v) + F_r(u,v)$.
- Homomorphic Filter Transfer Function: $H(u,v) = (\gamma_H - \gamma_L)[1 - e^{-c(D^2(u,v)/D_0^2)}] + \gamma_L$, with $\gamma_L < 1$ and $\gamma_H > 1$.
- Exponential Output: Reconstructed image $g(x,y) = \exp(\mathcal{F}^{-1}\{H(u,v) \cdot Z(u,v)\})$.

Color Image Enhancement Techniques

Extends spatial and frequency domain enhancement methods to multi-channel color representations.

- Pseudo-Color Processing: Assigns colors to gray levels or band ranges based on intensity thresholding.
- Full-Color Processing
  - Processes color image channels directly in RGB, HSI, or Lab color spaces.
  - RGB Processing: Modifying Red, Green, and Blue channels separately can alter original hue.
- HSI Color Model: Decouples Intensity (I) from color info (Hue H and Saturation S).
- Intensity Channel Enhancement: Applying contrast stretching or histogram equalization only on Intensity channel preserves true color hues.

AKTU Exam Practice Questions (Unit 2)

- **Q1** Explain histogram equalization step-by-step. Given a 3-bit image histogram, compute equalized gray level mappings.
- **Q2** Derive expressions for 2D Discrete Fourier Transform (DFT) and its inverse. Discuss spatial frequency representation.
- **Q3** Compare Ideal, Butterworth, and Gaussian Lowpass filters in frequency domain. Why does Ideal Lowpass cause ringing?
- **Q4** Explain Homomorphic filtering with a neat block diagram. How does it separate illumination and reflectance components?
- **Q5** Differentiate between Spatial Domain and Frequency Domain image enhancement techniques with suitable examples.
- **Q6** Explain first-order and second-order derivative spatial filters for image sharpening. Define Sobel and Laplacian masks.
- **Q7** What is Pseudo-color image processing? Explain intensity slicing and color transformation methods.

Unit 2 done. You are exam-ready.

Revise the Exam corner once more, then open Unit 3 on theycotes.com. Every AKTU subject, unit by unit, free. Notes by Chirag Jain, Ycotes.

