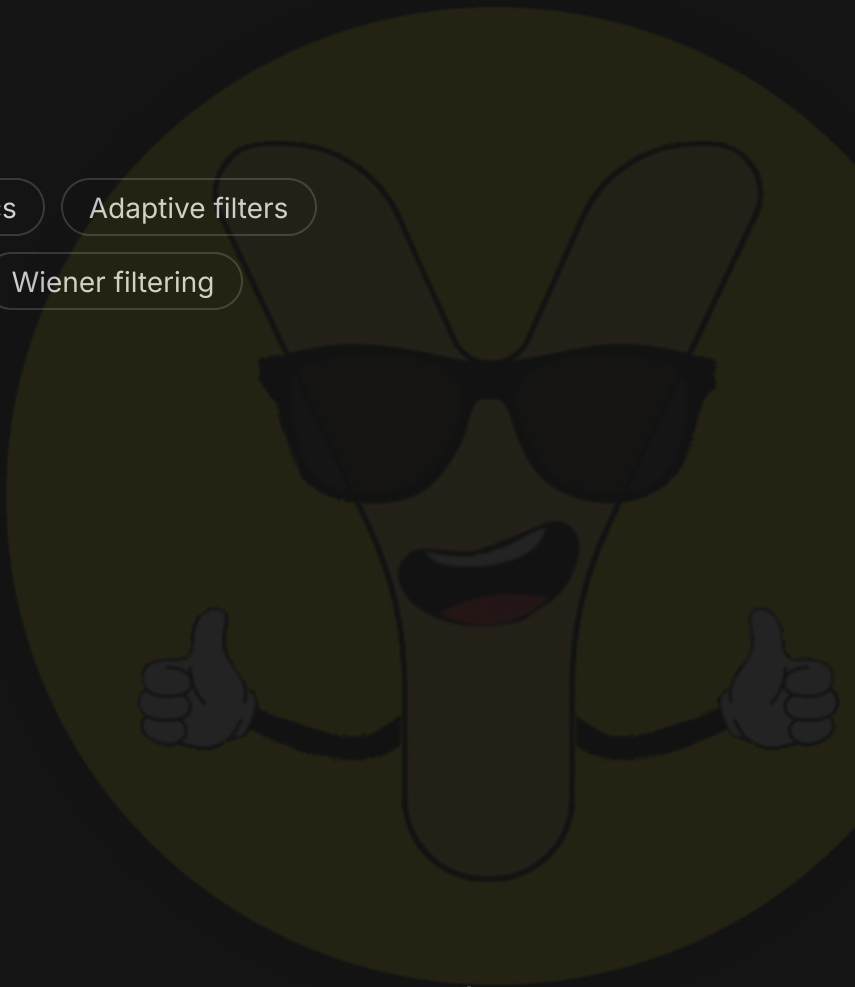
AKTU · 3RD YEAR · SEMESTER 5 · BCS057

Image Processing

Unit 3 — Image Restoration

Image Restoration | Degradation model | Properties | Noise models | Mean Filters | Order Statistics | Adaptive filters | Band reject Filters | Band pass Filters | Notch Filters | Optimum Notch Filtering | Inverse Filtering | Wiener filtering



DECODED BY

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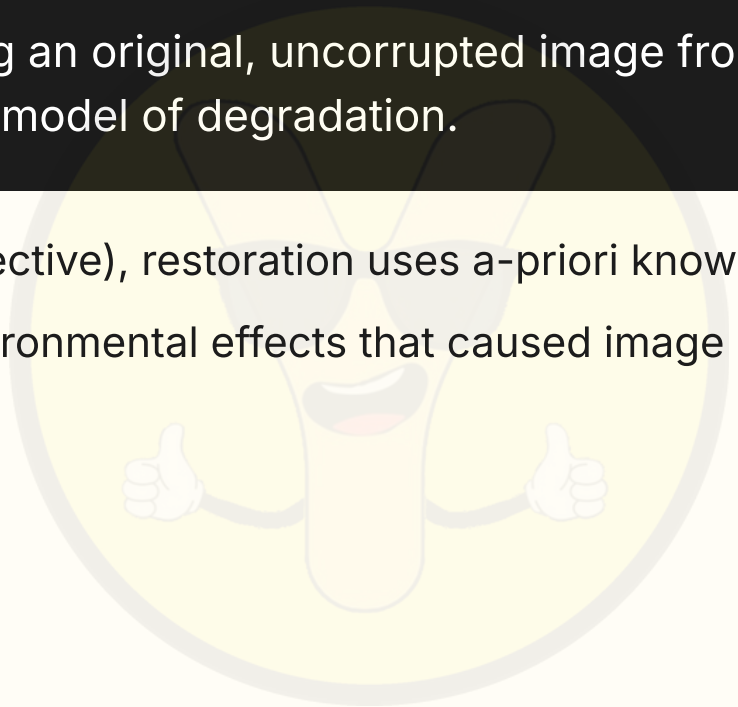
Image Restoration

IMAGE RESTORATION

An objective process of recovering an original, uncorrupted image from a degraded and noisy observation using a mathematical model of degradation.

- Unlike enhancement (which is subjective), restoration uses a-priori knowledge of degradation.
- Aims to reverse the physical or environmental effects that caused image blurring and noise.



Degradation Model

IMAGING DEGRADATION MODEL

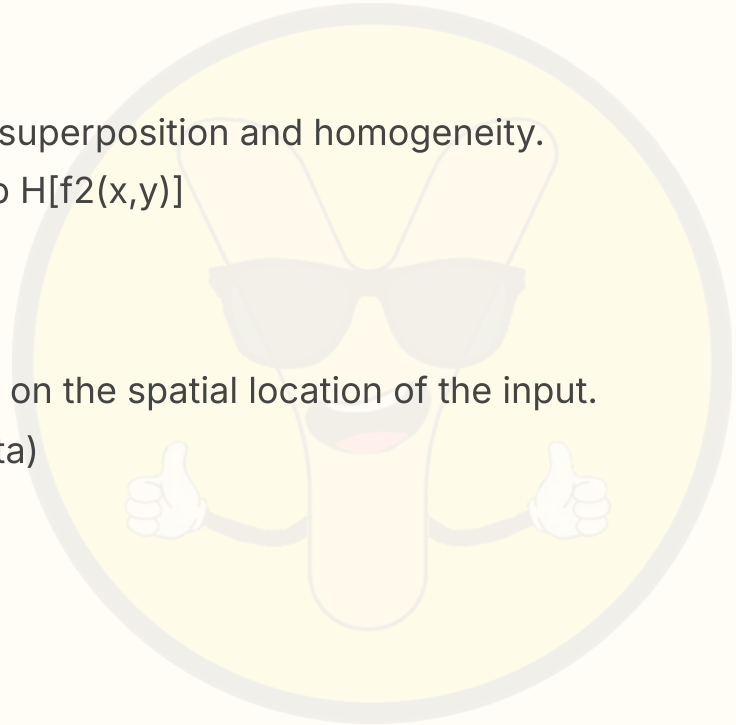
A mathematical representation combining a degradation function (operator) and additive noise to describe image corruption.

- Represented in spatial domain as: $g(x, y) = H[f(x, y)] + \eta(x, y)$
- $f(x, y)$: original image, H: degradation operator, $\eta(x, y)$: additive noise term.

Properties of Degradation Model

Key assumptions that simplify the mathematical restoration process.

- Linearity
  - The degradation operator must satisfy superposition and homogeneity.
  - H[a f1(x,y) + b f2(x,y)] = a H[f1(x,y)] + b H[f2(x,y)]
- Position Invariance
  - Also known as Shift-Invariance.
  - The system response does not depend on the spatial location of the input.
  - H[f(x-alpha, y-beta)] = g(x-alpha, y-beta)



Noise Models

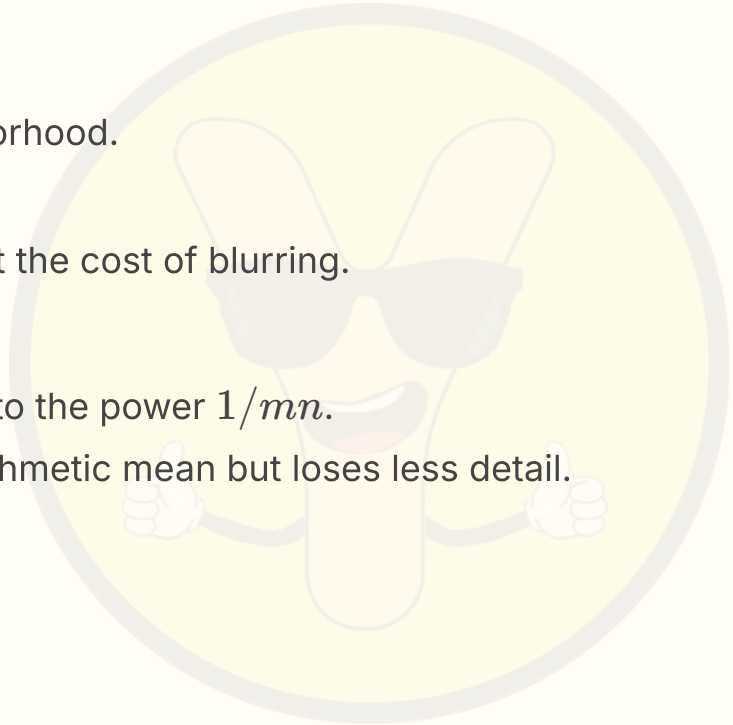
Common statistical distributions used to model spatial noise in images.

- Gaussian Noise
  - Arises from electronic circuit noise and sensor thermal noise.
  - Probability Density Function (PDF) follows a Gaussian bell curve.
- Impulse (Salt-and-Pepper) Noise
  - Appears as sharp, sudden disturbances like bright white or dark black pixels.
  - Caused by malfunctioning pixels, faulty memory locations, or transmission sync errors.

Spatial Mean Filters

Smoothing filters used primarily to reduce noise by local averaging.

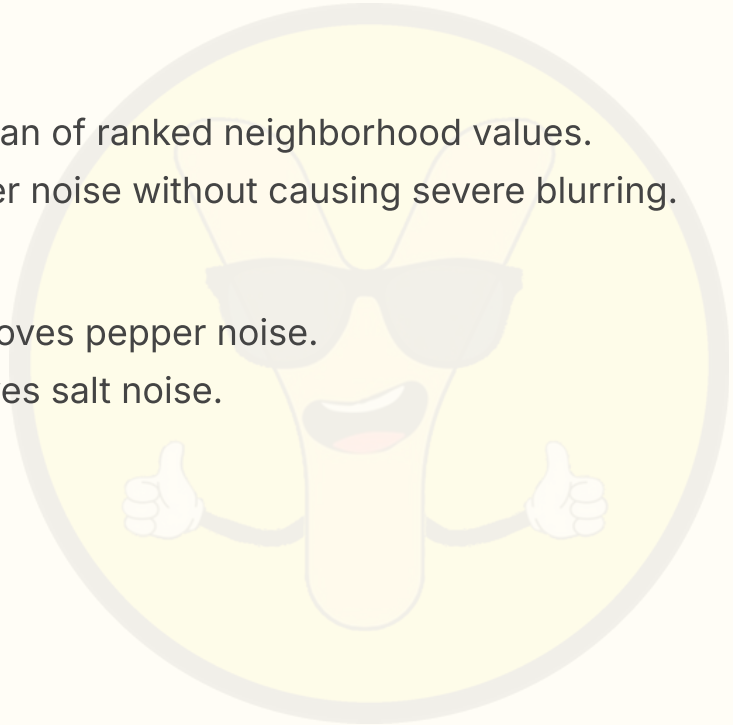
- Arithmetic Mean Filter
  - Computes average value in the neighborhood.
  - $\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$
  - Reduces Gaussian and uniform noise at the cost of blurring.
- Geometric Mean Filter
  - Computes the product of pixels raised to the power $1/mn$.
  - Achieves smoothing comparable to arithmetic mean but loses less detail.



Order-Statistics Filters

Spatial filters based on ranking pixel values in a neighborhood window.

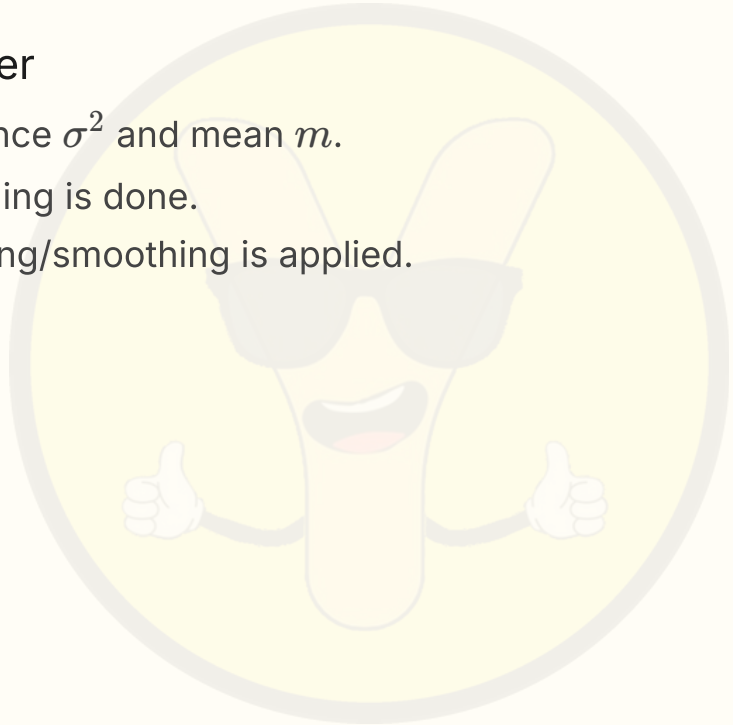
- Median Filter
  - Replaces the center pixel with the median of ranked neighborhood values.
  - Highly effective against salt-and-pepper noise without causing severe blurring.
- Max and Min Filters
  - Max filter finds the brightest point, removes pepper noise.
  - Min filter finds the darkest point, removes salt noise.



Adaptive Filters

Filters that change behavior based on local image characteristics.

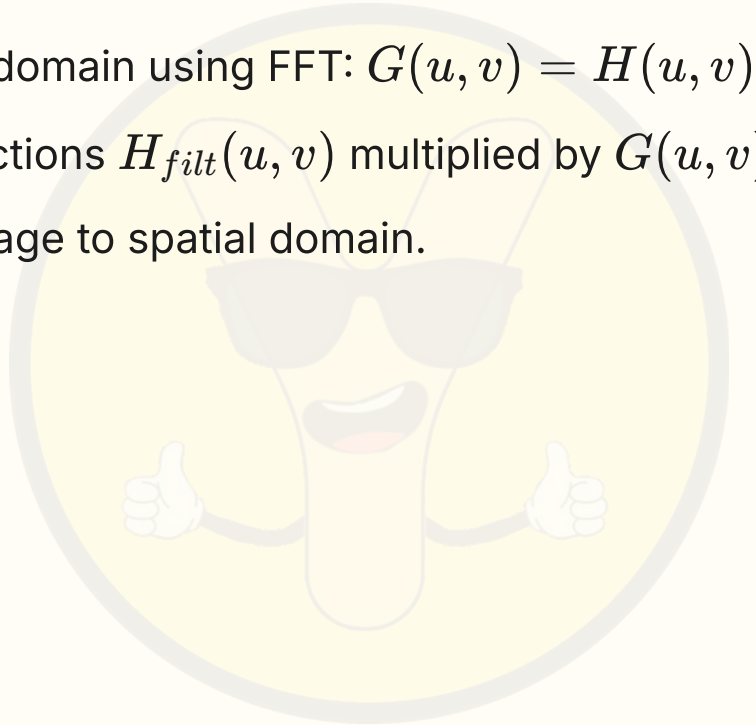
- Adaptive Local Noise Reduction Filter
  - Behavior changes based on local variance σ^2 and mean m.
  - If variance is high (edges), little smoothing is done.
  - If variance is low (smooth areas), blurring/smoothing is applied.



Frequency Domain Filtering

Restoration using the 2D Fourier Transform to remove periodic or targeted noise.

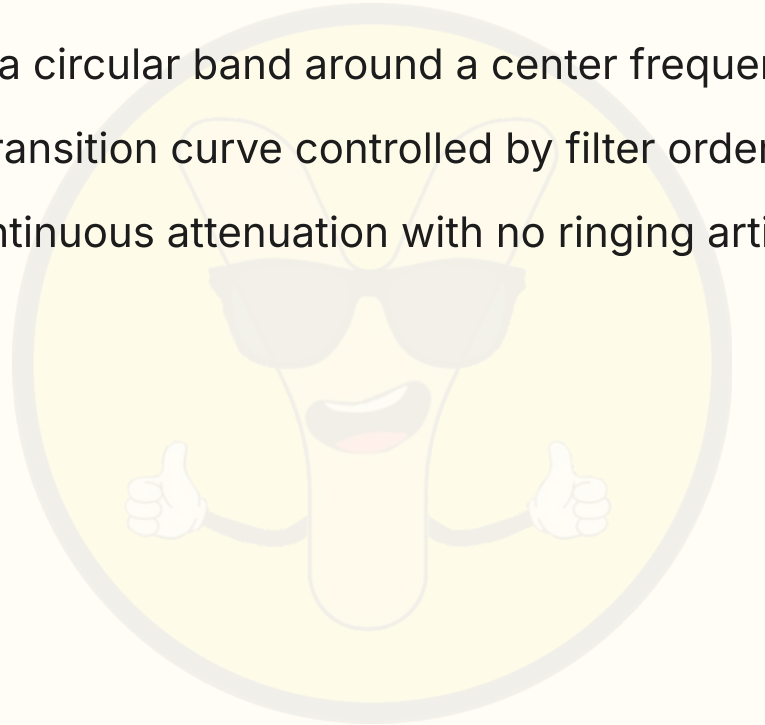
- Image is transformed to frequency domain using FFT: $G(u,v) = H(u,v)F(u,v) + N(u,v)$.
- Filters are designed as transfer functions $H_{filt}(u,v)$ multiplied by $G(u,v)$.
- Inverse FFT returns the restored image to spatial domain.



Band-Reject Filters

Frequency domain filters that remove a specific band of frequencies.

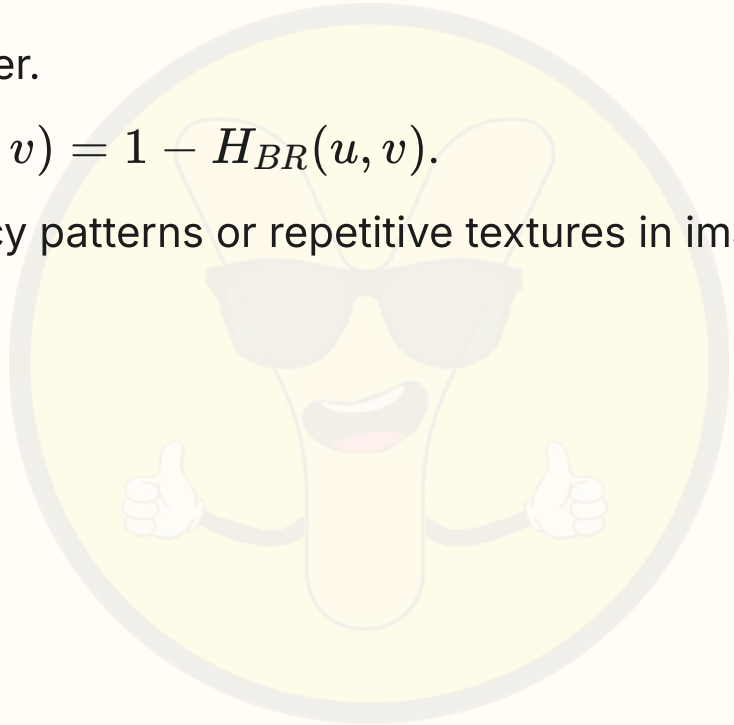
- Ideal Band-Reject: Sharply cuts off a circular band around a center frequency.
- Butterworth Band-Reject: Smooth transition curve controlled by filter order n.
- Gaussian Band-Reject: Smooth, continuous attenuation with no ringing artifacts.



Band-Pass Filters

Filters that allow a specific band of frequencies to pass while rejecting others.

- Complement of the Band-Reject filter.
- Mathematically defined as: $H_{BP}(u, v) = 1 - H_{BR}(u, v)$.
- Useful in isolating specific frequency patterns or repetitive textures in images.



Notch Filters

NOTCH FILTER

A frequency domain filter that rejects (or passes) frequencies in targeted, localized neighborhoods around specific frequency coordinates.

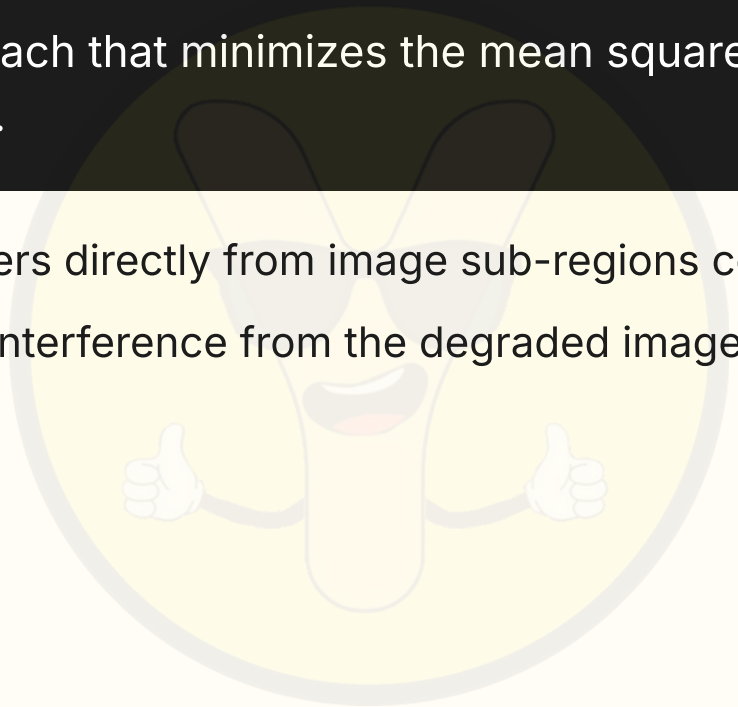
- Extremely useful for removing periodic noise patterns appearing as spikes in the Fourier spectrum.
- Must be implemented in pairs symmetric to the origin to maintain real-valued spatial images.

Optimum Notch Filtering

OPTIMUM NOTCH FILTERING

A specialized notch filtering approach that minimizes the mean square error between the estimated noise pattern and the actual noise.

- Estimates sinusoidal noise parameters directly from image sub-regions containing only noise.
- Subtracts the estimated sinusoidal interference from the degraded image spectrum.

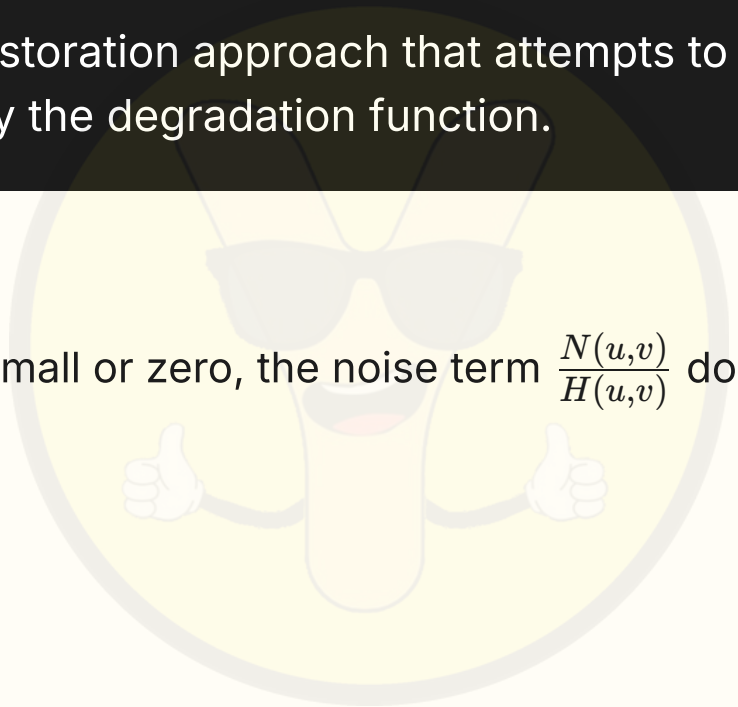


Inverse Filtering

INVERSE FILTERING

The simplest frequency domain restoration approach that attempts to reverse the degradation by dividing the degraded spectrum by the degradation function.

- Formula: $\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$
- Major limitation: If $H(u,v)$ is very small or zero, the noise term $\frac{N(u,v)}{H(u,v)}$ dominates and destroys the result.



Wiener Filtering

WIENER FILTERING (MINIMUM MEAN SQUARE ERROR)

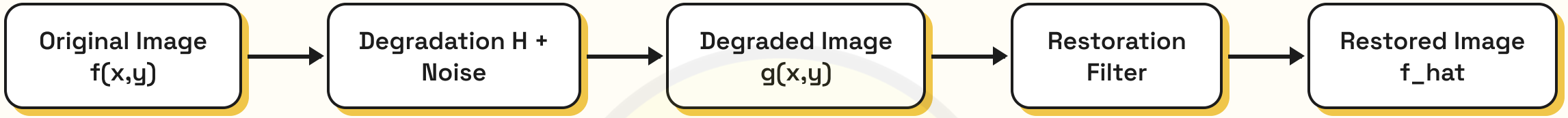
An optimal restoration filter that balances inverse filtering and noise smoothing based on statistical measures of image and noise.

- Minimizes the mean square error between estimated image and original image.
- Formula incorporates power spectra: $\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}\right] G(u,v)$

Inverse Filtering vs. Wiener Filtering

Feature	Inverse Filtering	Wiener Filtering
Noise Handling	Amplifies noise heavily	Suppresses noise optimally
Optimization	None (Direct division)	Minimizes Mean Square Error
Prior Knowledge	Requires only H(u,v)	Requires noise and image power spectra
Result Quality	Poor if H(u,v) is small	Produces clear, balanced output

Degradation and Restoration Flow



End-to-end pipeline from image formation, degradation, to restoration.

Exam corner

- **Q1** Explain the degradation model for continuous and digital images with mathematical formulations.
- **Q2** Compare spatial mean filters and order-statistics filters with suitable mathematical definitions.
- **Q3** What is inverse filtering? Discuss its primary limitation and how Wiener filtering overcomes it.
- **Q4** Explain the working principle and mathematical expression of the Adaptive Local Noise Reduction Filter.
- **Q5** Write short notes on: (a) Notch Filters (b) Band-Reject Filters.
- **Q6** Derive the Wiener filter equation and explain its components in the frequency domain.

Unit 3 done. You are exam-ready.

Revise the Exam corner once more, then open Unit 4 on theycotes.com. Every AKTU subject, unit by unit, free. Notes by Chirag Jain, Ycotes.

