UNIT–3 : Image Restoration (PYQ Solution)

AKTU · Image Processing (BCS057) · Solutions by **Chirag Jain**, Ycotes

SECTION A Answers (≈70 Words Each)

Q1. Define the advantages of the Wiener filter. Compare the inverse filter with the Wiener filter. (2023)

The Wiener filter minimises the mean square error between the original and restored image: $\hat{F} = [H / (|H|^2 + K)] G$, *where K = Sn/Sf is the noise-to-signal power ratio. Advantages: it handles both blur and noise, avoids the noise amplification where H is small, and is optimal in the MSE sense. The inverse filter*, $\hat{F} = G / H$, ignores noise, so it blows up when H is near zero; Wiener needs power spectra knowledge.

Q2. Explain the term PSF. (2023)

The Point Spread Function (PSF) is the response of an imaging system to a point source of light, h(x, y). It describes how a single point is spread (blurred) by the system. The degraded image is the convolution $g = h * f + \eta$, so the PSF is the spatial form of the degradation function H(u, v) used in restoration.

Q3. What do you mean by degradation? (2023)

Degradation is any process that reduces the quality of an image during acquisition, transmission or storage, such as blur from motion or defocus, sensor noise and atmospheric turbulence. It is modelled as $g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$, where h is the degradation function and η the noise. Restoration tries to undo it.

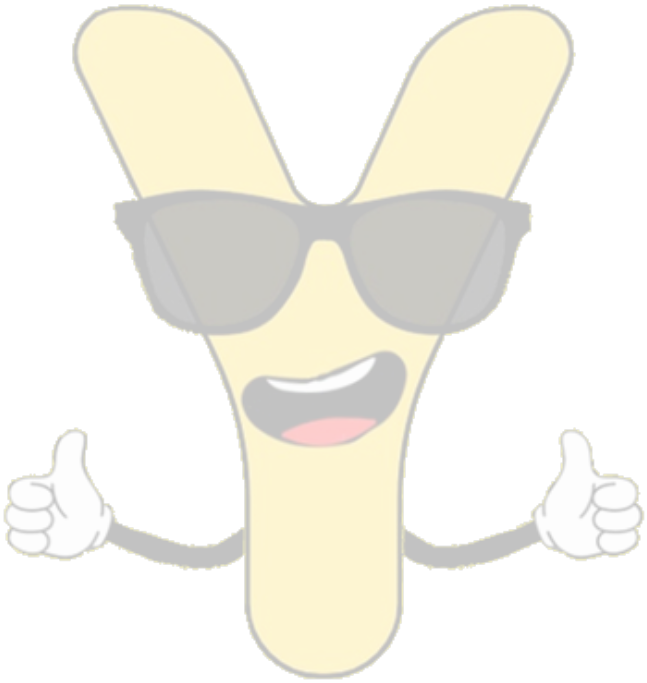
Q4. Define the harmonic mean filter. (2023)

The harmonic mean filter restores each pixel as $\hat{f}(x, y) = mn / \Sigma (1 / g(s, t))$ over an m × n neighbourhood. It works well for salt noise and Gaussian noise but

fails for pepper noise (zero values make the sum infinite). It is a non-linear averaging filter.

Q5. Differentiate between image enhancement and image restoration. (2019)

Enhancement is subjective: it improves appearance (contrast, sharpness) without a degradation model. Restoration is objective: it reverses a known degradation using a model of blur and noise to recover the original image. Enhancement uses heuristics like histogram equalisation; restoration uses inverse and Wiener filtering.



SECTION B Answers (≈250 Words Each)

Q1. Explain the band-pass filter technique for noise reduction. (2023)

Periodic noise (from electrical or electromechanical interference) appears as concentrated bursts of energy at specific frequencies in the Fourier spectrum. Frequency-domain **band filters** remove it.

Band-reject filter: removes a band of frequencies around radius D0 with width W and passes the rest:

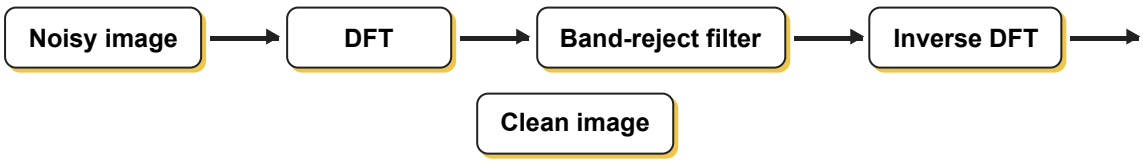
- Ideal: H = 0 if D0 − W/2 ≤ D ≤ D0 + W/2, else 1.
- Butterworth: H = 1 / [1 + (D W / (D² − D0²))^(2n)].
- Gaussian: H = 1 − exp(−[(D² − D0²) / (D W)]²).

Band-pass filter: the opposite, H_bp = 1 − H_br; it passes only the selected band. It is used to **isolate the noise pattern** itself: the noise pattern η(x, y) = F^{-1}{H_bp(u, v) G(u, v)} is extracted, and the cleaned image is $\hat{f}$ = g − w(x, y) η(x, y), where the weight w minimises the local variance of $\hat{f}$.

Notch filters are narrow band filters that reject or pass small regions around chosen frequencies and their symmetric counterparts.

Procedure:

1. Compute the DFT of the noisy image and view its spectrum.
2. Locate the noise spikes or rings.
3. Design the band-reject (or notch) filter around them.
4. Multiply and take the inverse DFT.



Band filtering for periodic noise

Conclusion: band filters remove periodic noise without blurring the rest of the image.

Q2. Discuss the noise models present in an image. How can they be

removed using spatial filtering? (2019)

Noise is random variation in pixel values, modelled by its probability density function (PDF), usually independent of the image and additive: $g = f + \eta$.

Noise models:

- **Gaussian:** $p(z) = (1/(\sqrt{(2\pi)}\,\sigma))\exp(-(z-\mu)^2/2\sigma^2)$; caused by electronic circuit noise and sensor heat.
- **Rayleigh:** skewed PDF, used in range imaging.
- **Erlang (gamma) and exponential:** laser imaging.
- **Uniform:** equal probability over a range; used in simulations.
- **Impulse (salt-and-pepper):** isolated black (pepper) and white (salt) pixels, from faulty switching or transmission errors.

Removal using spatial filters:

Noise	Filter
Gaussian, uniform	**Arithmetic mean** ($1/mn\ \Sigma\ g$), **geometric mean**, Gaussian smoothing
Salt-and-pepper	**Median filter** (middle value of the window)
Salt only	Harmonic mean, min filter, contra-harmonic with $Q < 0$
Pepper only	Max filter, contra-harmonic with $Q > 0$
Both impulses	**Alpha-trimmed mean**, adaptive median

- The **midpoint** filter = (max + min)/2 suits Gaussian and uniform noise.
- **Adaptive filters** change behaviour with local variance and preserve edges better.

Example: the window 12 14 200 13 15 has median 14, removing the impulse 200, whereas the mean (50.8) would smear it.

Conclusion: the filter choice must match the noise model.

SECTION C Answers (≈500 Words Each)

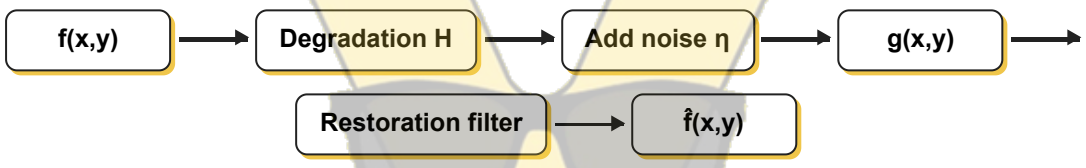
Q1. What is image restoration? Draw and explain the basic block diagram of the restoration process. Discuss the degradation model and restoration by inverse filtering. (2023, 2019)

Image restoration is the process of recovering an original image from a degraded one using prior knowledge of the degradation phenomenon. It is objective: it models the degradation and applies the inverse process.

Degradation model: the original f(x, y) passes through a degradation function h(x, y) and noise η(x, y) is added:

$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$

In the frequency domain, **$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$**.



Degradation and restoration model

Explanation of the blocks:

- **Degradation function H:** blur such as motion, defocus or turbulence.
- **Noise η:** additive random noise (Gaussian, impulse).
- **Restoration filter:** estimates $\hat{f}(x, y)$ as close to f as possible using knowledge of H and η.

Inverse filtering: the simplest method ignores noise and divides by the degradation function:

$\hat{F}(u, v) = G(u, v) / H(u, v) = F(u, v) + N(u, v)/H(u, v)$

Then $\hat{f} = F^{-1}\{\hat{F}\}$.

Steps:

1. Compute the DFT of the degraded image g.
2. Estimate H(u, v) (by observation, experiment or modelling).
3. Divide G by H.
4. Take the inverse DFT to obtain $\hat{f}$.

Limitations:

- Even when H is known exactly, the term N/H cannot be removed since N is unknown.
- Where H(u, v) is zero or very small, **N/H dominates** and amplifies noise, ruining the result.
- Remedy: limit the division to frequencies near the origin (**pseudo-inverse**) or use the **Wiener filter**, $\hat{F} = [|H|^2 / (|H|^2 + K)] (G / H)$, which accounts for noise.

Example: with H = 0.01 at a high frequency and noise N = 0.5, the error is N/H = 50, far larger than the signal.

Conclusion: inverse filtering restores noise-free blur well but is unstable with noise, so Wiener filtering is preferred in practice.

Q2. Explain the median filter and the midpoint filter. (2023)

Both are **order-statistics filters**, which rank the pixels in the window Sxy and select a value.

1. Median filter: $\hat{f}(x, y)$ = median over (s, t) in Sxy of g(s, t). The pixel is replaced by the middle value of the sorted neighbourhood.

- Excellent for **salt-and-pepper (impulse) noise**; it removes isolated spikes while **preserving edges** and causing much less blur than the mean filter.
- Example (3 × 3 window): values 10, 12, 11, 255, 13, 12, 10, 11, 12. Sorted: 10, 10, 11, 11, 12, 12, 12, 13, 255. Median = **12**, so the outlier 255 is removed.
- Variations: max filter (reduces pepper noise) and min filter (reduces salt noise).

2. Midpoint filter: $\hat{f}(x, y) = ½$ [max over Sxy of g(s, t) + min over Sxy of g(s, t)].

- It averages the largest and smallest values of the window, combining order statistics with averaging.
- Works best for **randomly distributed noise such as Gaussian and uniform noise**.
- Example for the same window: (255 + 10)/2 = **132.5**, showing it is not suitable when an outlier is present.

Basis	Median	Midpoint
Value chosen	Middle rank	(Max + Min)/2
Best noise	Salt and pepper	Gaussian, uniform
Linear	No	No
Edge effect	Preserves edges	Blurs more
Cost	Needs sorting	Only max and min



Order-statistics filtering

Conclusion: the median handles impulse noise; the midpoint handles random noise.

