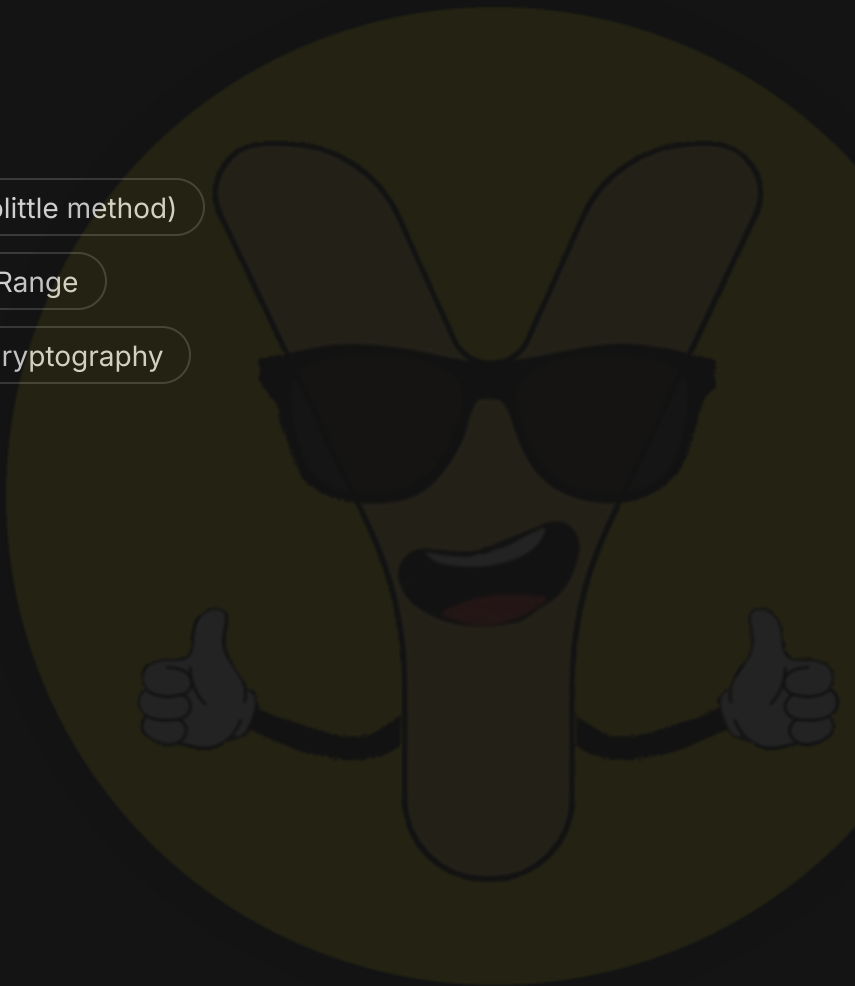
AKTU · 1ST YEAR · SEMESTER 2 · AAS203D

Numerical Methods

Unit 5 — Applied Linear Algebra

- Eigen Value Decomposition and Singular Value Decomposition
- Matrix factorization
- LU Decomposition (Dolittle method)
- QR Decomposition
- Linear transformation
- Matrix representation of Linear Transformation
- Kernel and Range
- Introduction to inner product space
- Norm of a vector
- Applications: 2D & 3D graphics, Neural Networks, Cryptography



DECODED BY

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Unit 5: Applied Linear Algebra

- Eigen Value Decomposition (EVD) and Singular Value Decomposition (SVD)
- Matrix Factorization techniques (LU and QR)
- Linear Transformations and Matrix Representations
- Kernel, Range, and Inner Product Spaces
- Vector Norms and Metrics
- Applications: Graphics, Neural Networks, and Cryptography



Eigen Value Decomposition (EVD)

EVD

For a square matrix A, EVD is the factorization $A = PDP^{-1}$, where D is a diagonal matrix of eigenvalues and P is the matrix of eigenvectors.

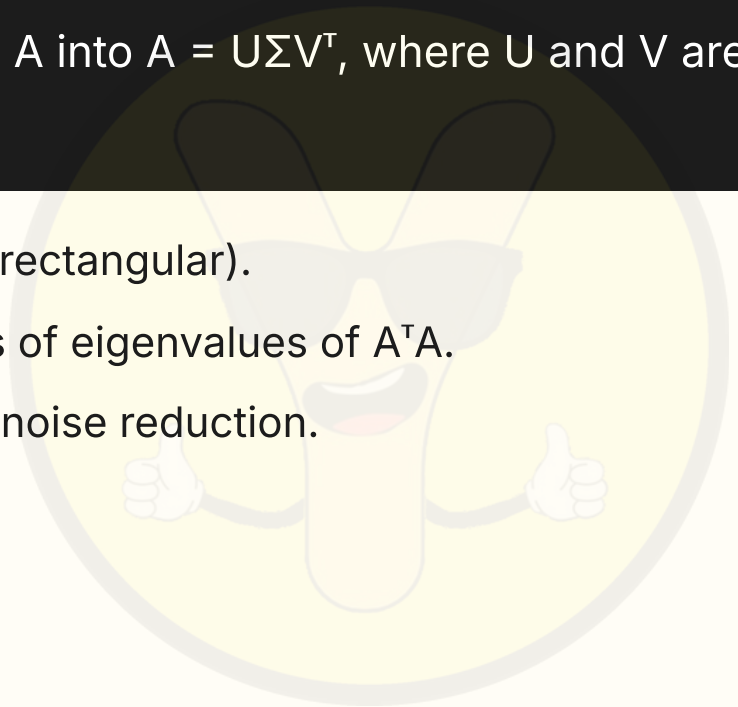
- Requirement: Matrix A must be diagonalizable.
- Characteristic equation: $\det(A - \lambda I) = 0$.
- Used for stability analysis and solving systems of linear differential equations.

Singular Value Decomposition (SVD)

SVD

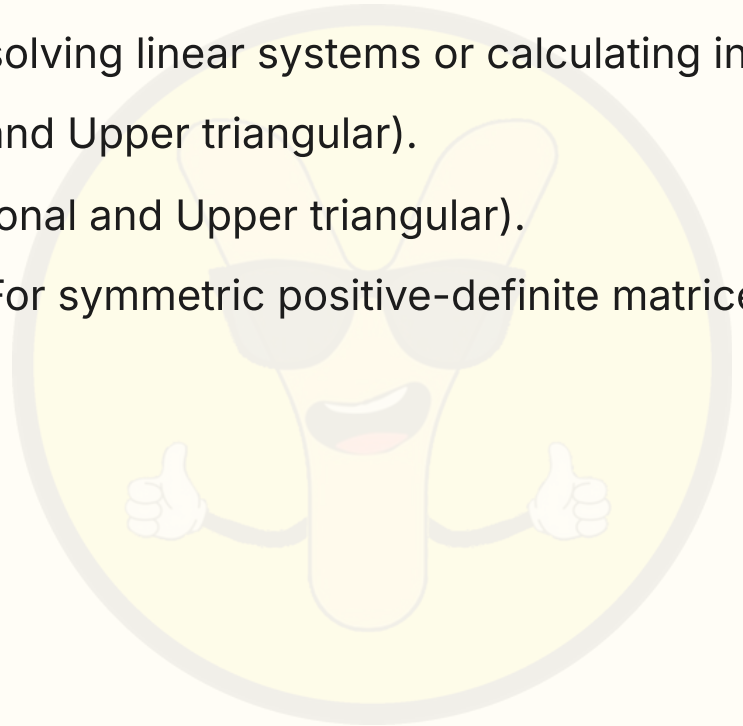
A factorization of any m x n matrix A into $A = U\Sigma V^T$, where U and V are orthogonal matrices and Σ is a diagonal matrix of singular values.

- Applicable to any matrix (square or rectangular).
- Singular values are the square roots of eigenvalues of A^TA.
- Essential for data compression and noise reduction.



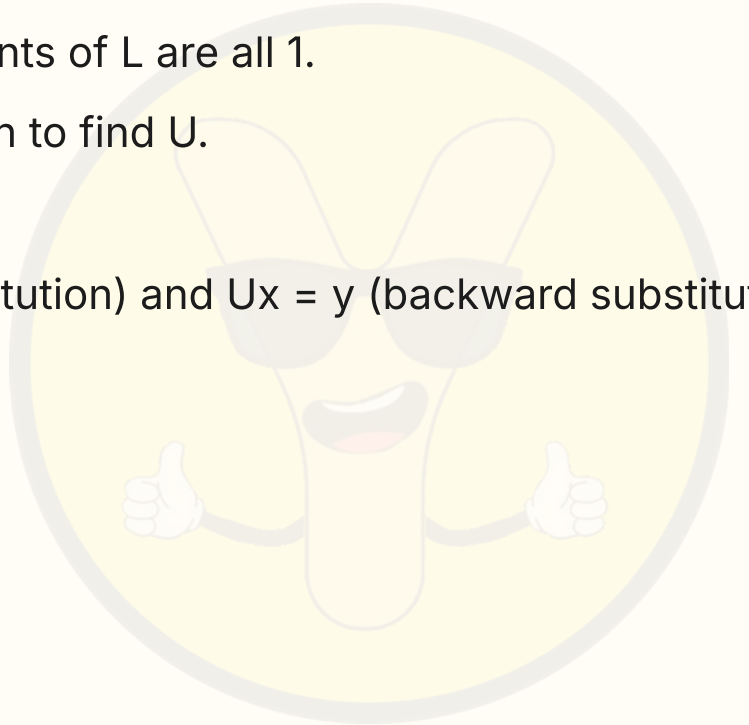
Matrix Factorization

- Process of decomposing a matrix into a product of simpler matrices.
- Simplifies complex operations like solving linear systems or calculating inverses.
- LU Decomposition: $A = LU$ (Lower and Upper triangular).
- QR Decomposition: $A = QR$ (Orthogonal and Upper triangular).
- Cholesky Decomposition: $A = LL^T$ (For symmetric positive-definite matrices).



LU Decomposition (Doolittle Method)

- Decomposes A into L (Lower) and U (Upper) such that A = LU.
- Doolittle constraint: Diagonal elements of L are all 1.
- Step 1: Perform Gaussian elimination to find U.
- Step 2: Store multipliers in L.
- Step 3: Solve Ly = b (forward substitution) and Ux = y (backward substitution).

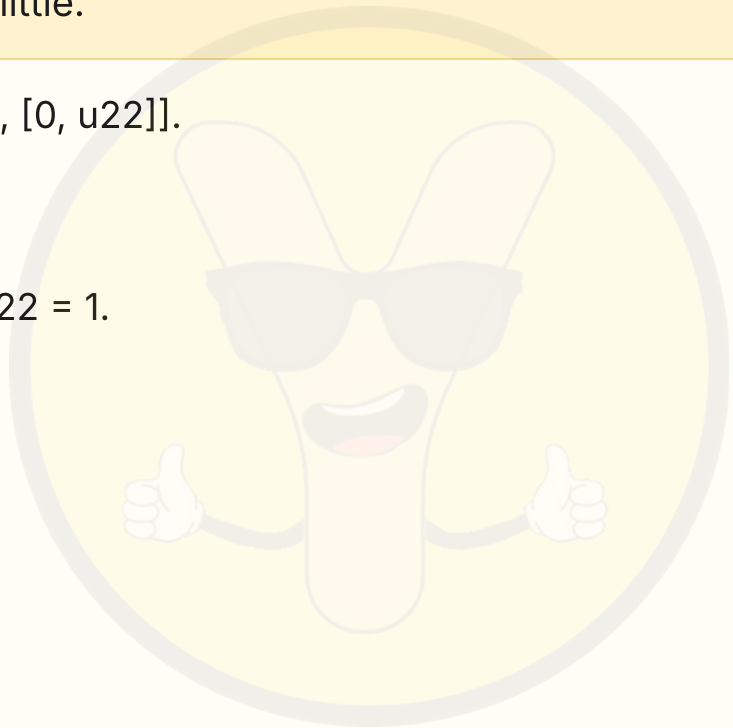


LU Decomposition Example

Decompose A = [[2, 1], [4, 3]] using Doolittle.

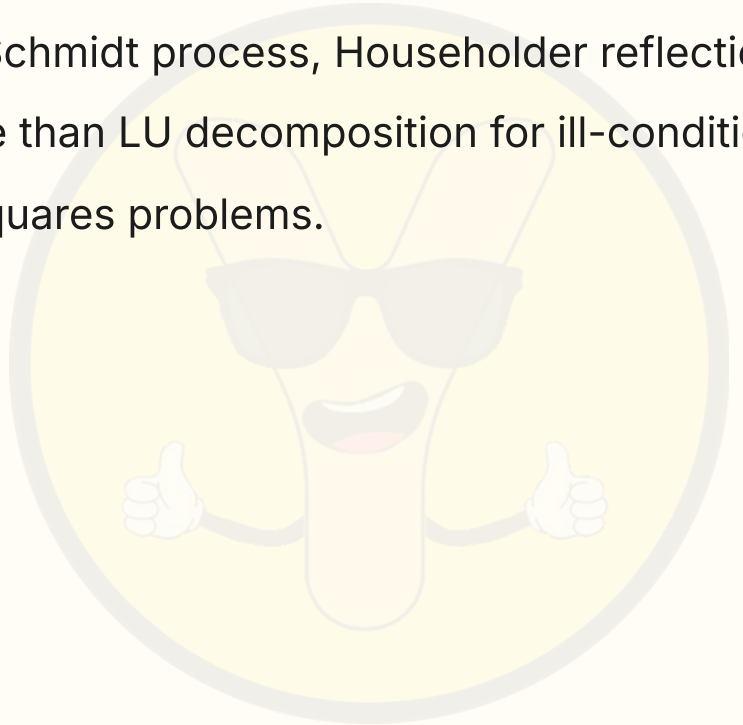
1. Set L = [[1, 0], [l21, 1]] and U = [[u11, u12], [0, u22]].
2. u11 = 2, u12 = 1.
3. l21 * u11 = 4 ⇒ l21 = 2.
4. l21 * u12 + u22 = 3 ⇒ 2(1) + u22 = 3 ⇒ u22 = 1.

L = [[1, 0], [2, 1]], U = [[2, 1], [0, 1]]



QR Decomposition

- Decomposes A into an orthogonal matrix Q ($Q^TQ = I$) and an upper triangular matrix R.
- Commonly computed using Gram-Schmidt process, Householder reflections, or Givens rotations.
- Advantage: More numerically stable than LU decomposition for ill-conditioned matrices.
- Used extensively in solving least-squares problems.

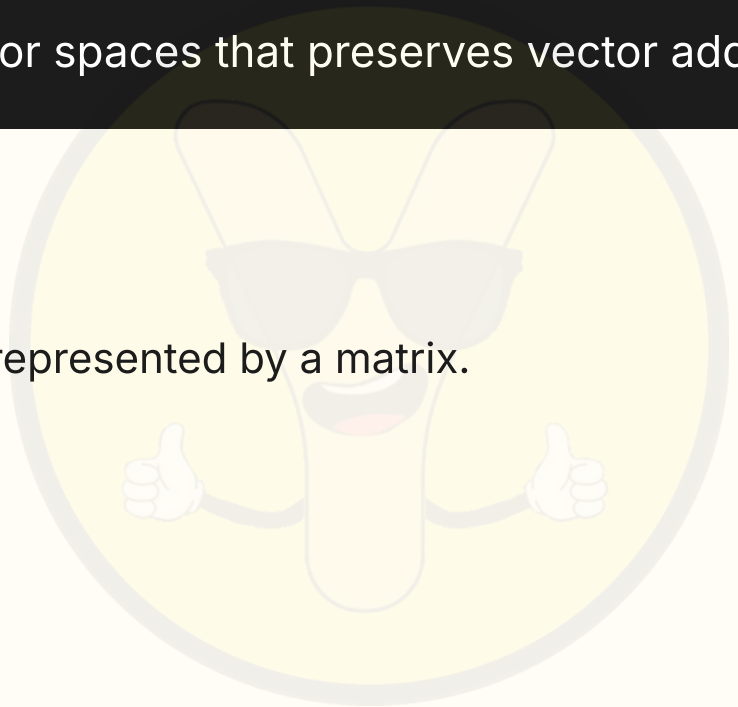


Linear Transformation

LINEAR MAP

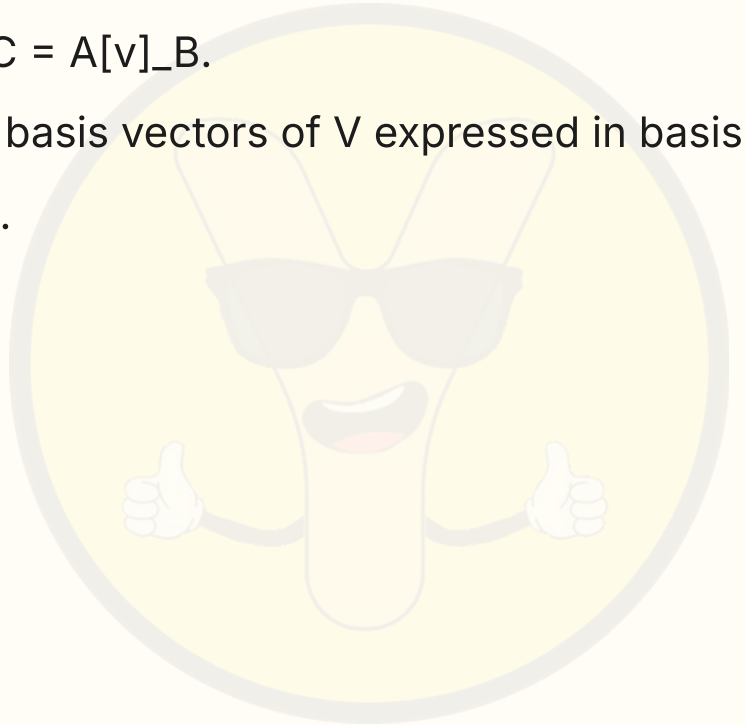
A function T: V → W between vector spaces that preserves vector addition and scalar multiplication.

- T(u + v) = T(u) + T(v)
- T(cu) = cT(u)
- Every linear transformation can be represented by a matrix.



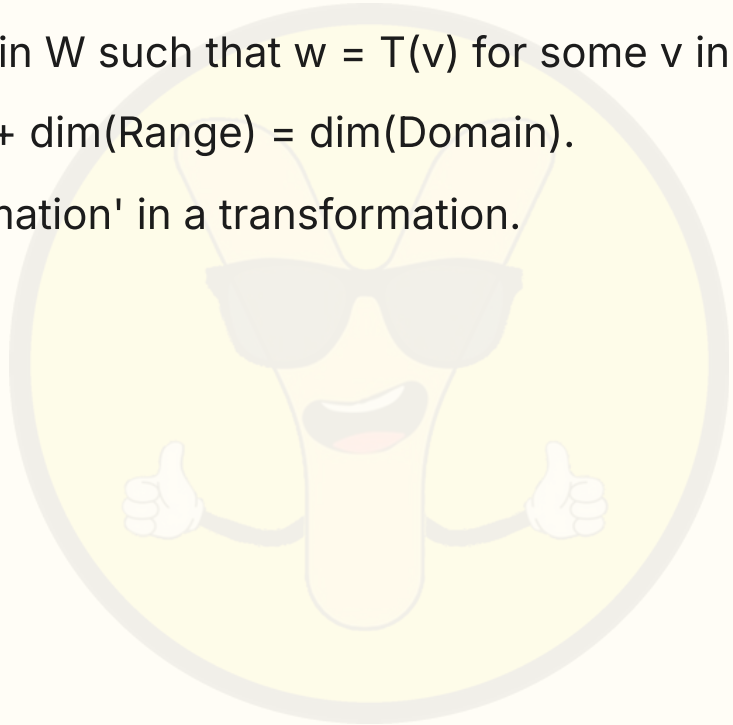
Matrix Representation

- Given a transformation T: V → W, choose bases B for V and C for W.
- The matrix A represents T if $[T(v)]_C = A[v]_B$.
- Columns of A are the images of the basis vectors of V expressed in basis C.
- Change of basis formula: $A' = P^{-1}AP$.



Kernel and Range

- Kernel (Null Space): Set of all vectors v such that T(v) = 0.
- Range (Image): Set of all vectors w in W such that w = T(v) for some v in V.
- Rank-Nullity Theorem: dim(Kernel) + dim(Range) = dim(Domain).
- Kernel represents the 'loss of information' in a transformation.

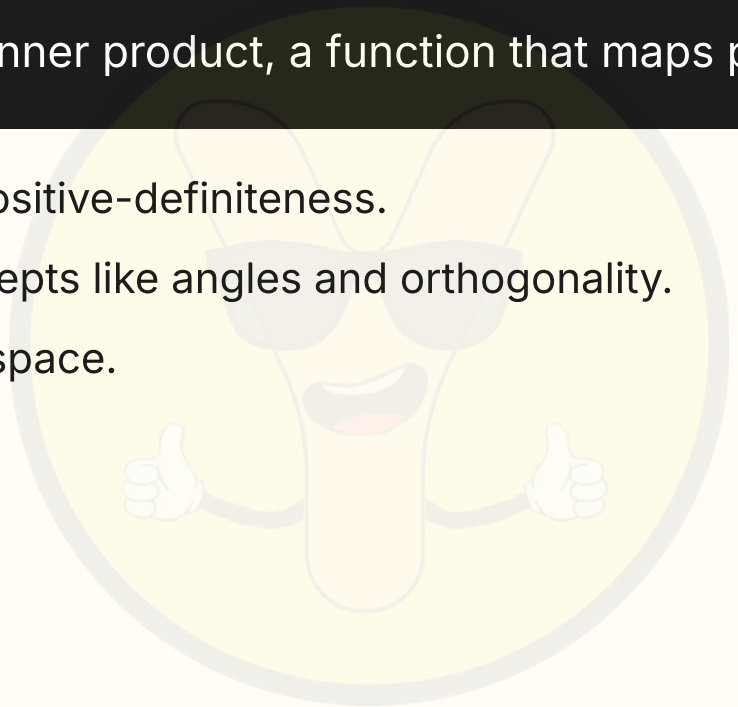


Inner Product Space

INNER PRODUCT

A vector space equipped with an inner product, a function that maps pairs of vectors to a scalar.

- Satisfies linearity, symmetry, and positive-definiteness.
- Allows definition of geometric concepts like angles and orthogonality.
- Example: Dot product in Euclidean space.

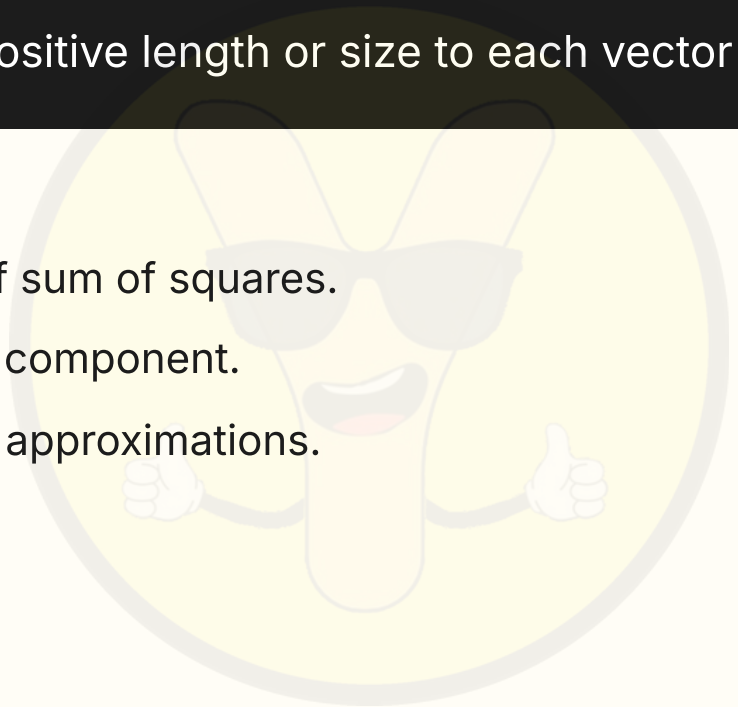


Norm of a Vector

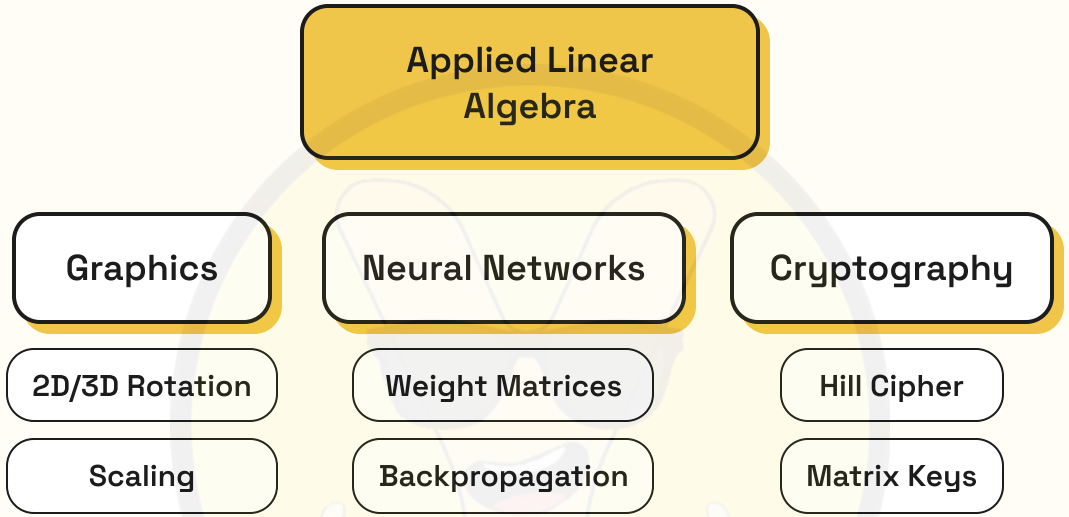
VECTOR NORM

A function that assigns a strictly positive length or size to each vector in a vector space.

- L1 Norm: Sum of absolute values.
- L2 Norm (Euclidean): Square root of sum of squares.
- L∞ Norm: Maximum absolute value component.
- Used to measure error in numerical approximations.



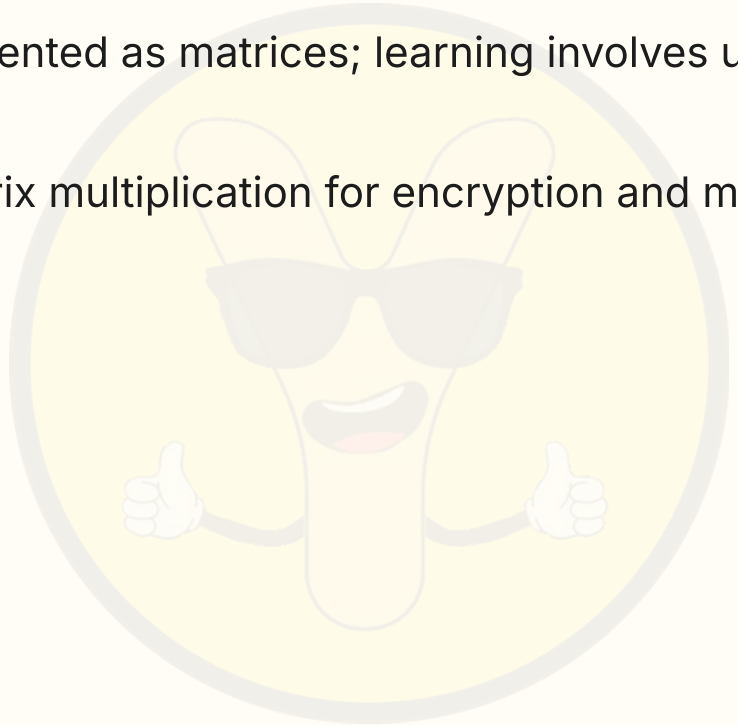
Applications Overview



Core applications of linear algebra in modern technology.

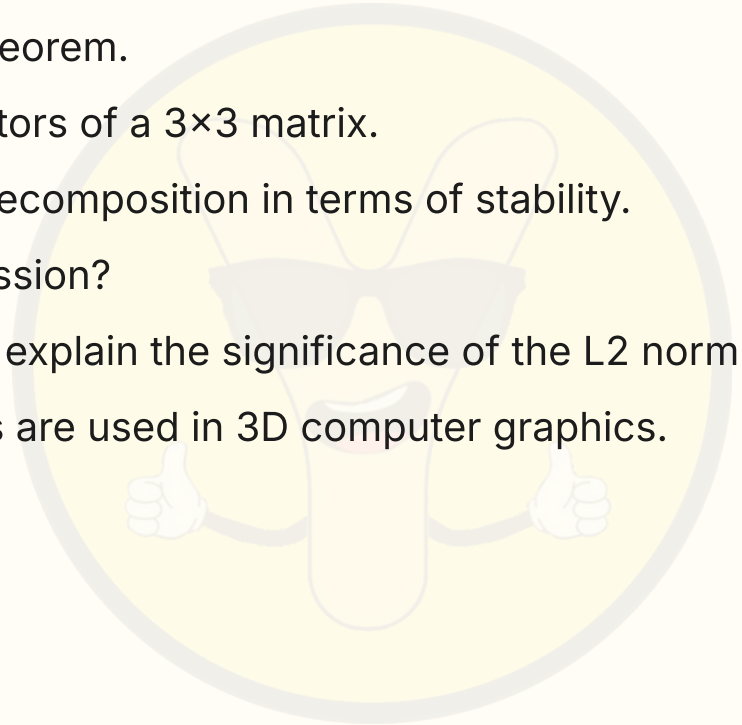
Applications Details

- Graphics: Transformations (Rotation, Scaling) are matrix multiplications.
- Neural Networks: Layers are represented as matrices; learning involves updating weights via matrix operations.
- Cryptography: Hill Cipher uses matrix multiplication for encryption and modular inverse for decryption.



Exam corner

- **Q1** Explain the Doolittle method for LU decomposition with a step-by-step algorithm.
- **Q2** State and prove the Rank-Nullity theorem.
- **Q3** Find the eigenvalues and eigenvectors of a 3×3 matrix.
- **Q4** Differentiate between LU and QR decomposition in terms of stability.
- **Q5** How is SVD used in image compression?
- **Q6** Define an inner product space and explain the significance of the L2 norm.
- **Q7** Explain how matrix transformations are used in 3D computer graphics.



Unit 5 done. You are exam-ready.

Revise the Exam corner once more, then open Unit 1 on theycotes.com. Every AKTU subject, unit by unit, free. Notes by Chirag Jain, Ycotes.

