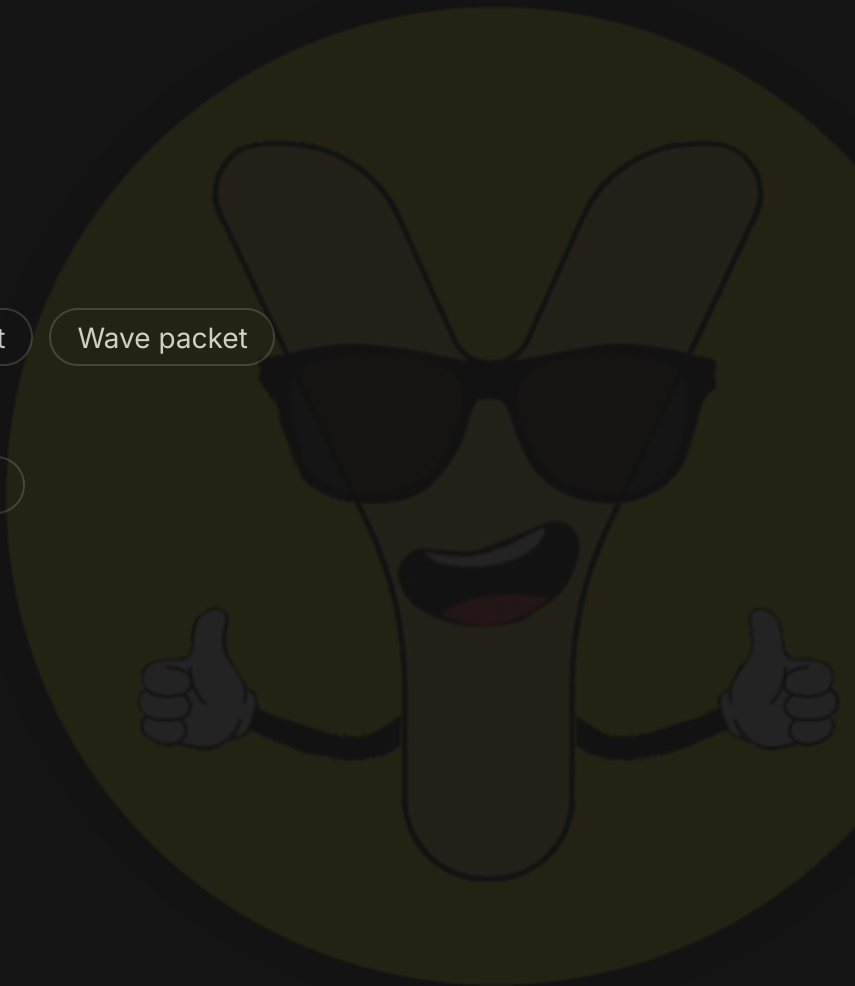
AKTU · 1ST YEAR · AAS101E / AAS201E

Quantum Physics and Applications

Unit 1 — Fundamentals of Quantum Mechanics

- Inadequacy of classical mechanics
- de-Broglie concept of matter waves
- Davisson and Germer experiment
- Wave packet
- Phase velocity
- Group velocity
- Heisenberg's uncertainty principle and its applications
- Time-dependent and time-independent Schrodinger wave equations
- Particle confined in one dimensional box
- Wave function and its physical interpretation



DECODED BY

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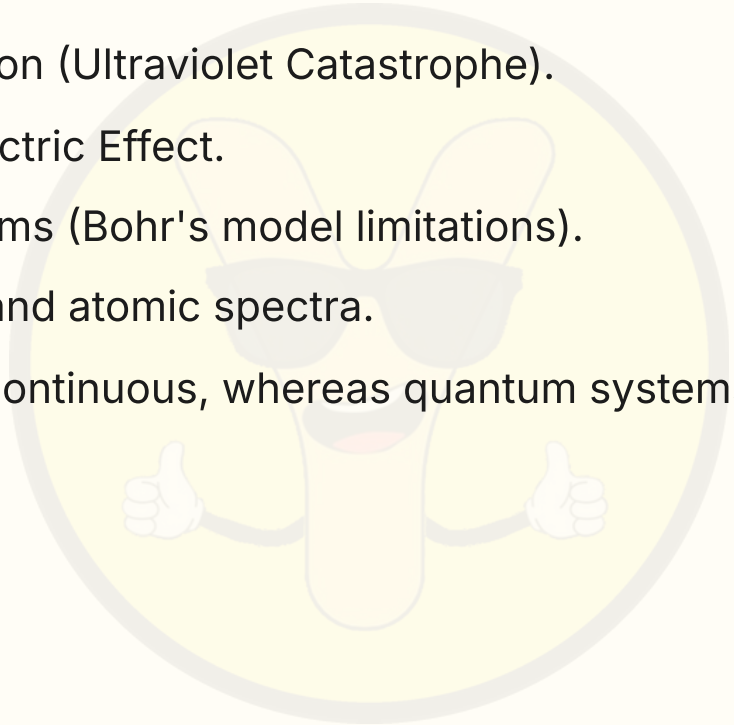
Founder, Ycotes · Gold Medalist in Artificial Intelligence, IIT Madras · taught 10,000+ students

Inadequacy of Classical Mechanics

Classical mechanics fails to explain phenomena at the atomic and subatomic scales.

- Failure to explain Blackbody Radiation (Ultraviolet Catastrophe).
- Inability to account for the Photoelectric Effect.
- Failure to explain the stability of atoms (Bohr's model limitations).
- Inability to explain Compton effect and atomic spectra.
- Classical physics treats energy as continuous, whereas quantum systems are quantized.



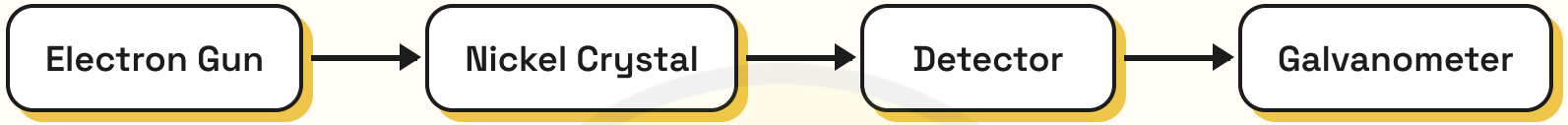
de-Broglie Concept of Matter Waves

MATTER WAVES

Louis de-Broglie proposed that every moving particle is associated with a wave, known as a matter wave.

- The wavelength is given by $\lambda = h/p$, where h is Planck's constant and p is momentum.
- This dual nature (wave-particle) is a fundamental postulate of quantum mechanics.
- The wavelength depends inversely on the mass and velocity of the particle.

Davisson and Germer Experiment



Experimental setup to demonstrate electron diffraction confirming de-Broglie hypothesis.

Davisson and Germer Experiment: Key Findings

The experiment provided the first experimental evidence for the wave nature of electrons.

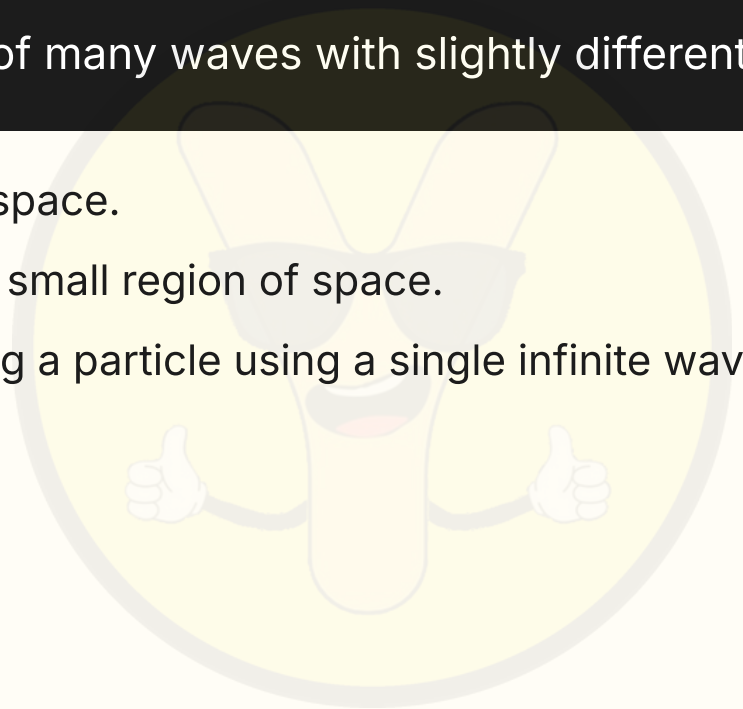
- Electrons were accelerated by a potential V and directed at a Nickel crystal.
- The scattered electrons showed a diffraction pattern similar to X-rays.
- The experimental wavelength matched the de-Broglie wavelength formula.
- This confirmed the wave-particle duality for matter particles.

Wave Packet

WAVE PACKET

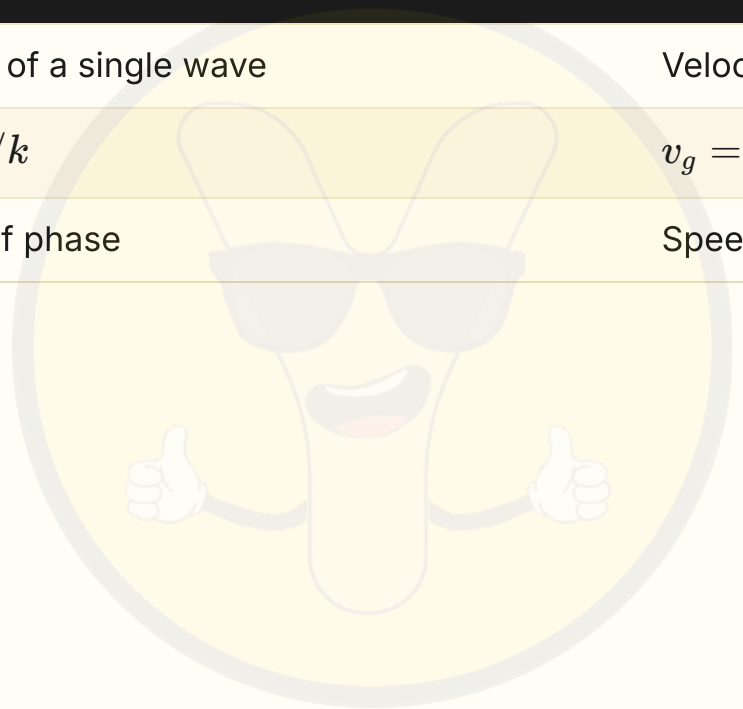
A wave packet is a superposition of many waves with slightly different frequencies and wavelengths.

- It represents a localized particle in space.
- The amplitude is non-zero only in a small region of space.
- It solves the problem of representing a particle using a single infinite wave.



Phase Velocity vs Group Velocity

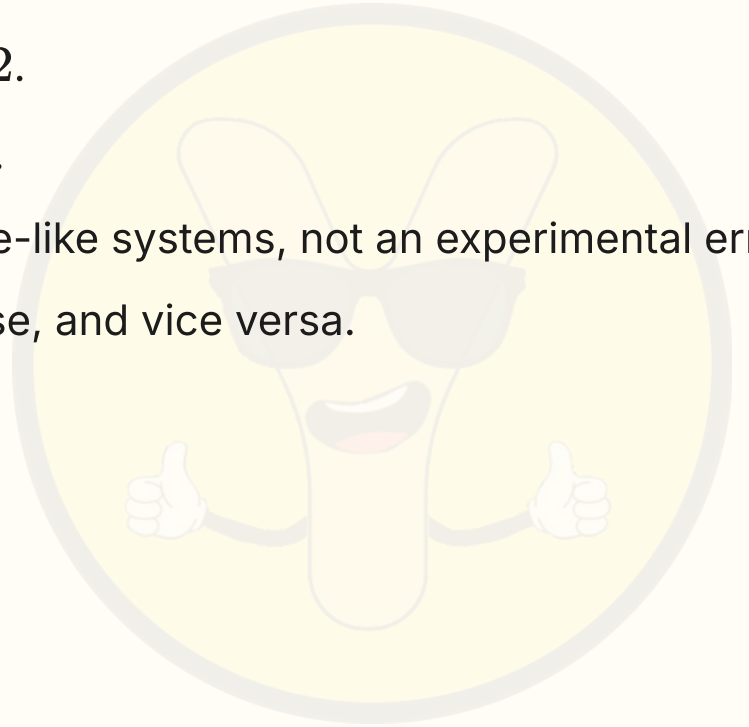
Feature	Phase Velocity (v_p)	Group Velocity (v_g)
Definition	Velocity of a single wave	Velocity of the wave packet
Formula	$v_p = \omega/k$	$v_g = d\omega/dk$
Physical meaning	Speed of phase	Speed of particle/energy



Heisenberg's Uncertainty Principle

It is impossible to simultaneously measure the exact position and momentum of a particle.

- Mathematical form: $\Delta x \cdot \Delta p \geq \hbar/2$.
- Energy-time form: $\Delta E \cdot \Delta t \geq \hbar/2$.
- It is a fundamental property of wave-like systems, not an experimental error.
- As Δx decreases, Δp must increase, and vice versa.



Application of Uncertainty Principle

Estimate the minimum energy of an electron confined in a box of width 1 Angstrom.

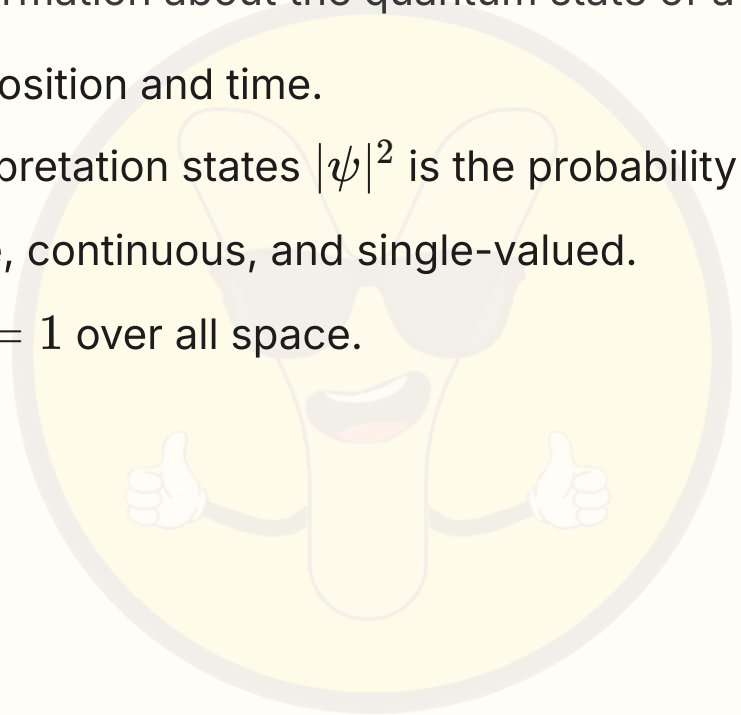
1. Assume $\Delta x = 10^{-10} m$.
2. Use $\Delta p \approx \hbar / \Delta x$.
3. Calculate kinetic energy $E = p^2 / 2m$.

Energy is approximately 3.8 eV.

Wave Function (ψ)

The wave function contains all the information about the quantum state of a system.

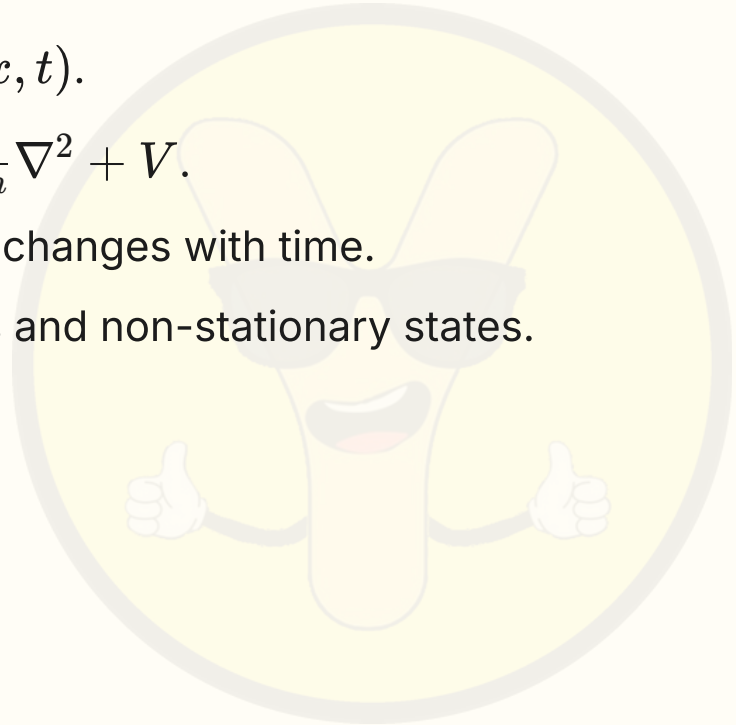
- It is a complex-valued function of position and time.
- Physical interpretation: Born's interpretation states $|\psi|^2$ is the probability density.
- A valid wave function must be finite, continuous, and single-valued.
- Normalization condition: $\int |\psi|^2 d\tau = 1$ over all space.



Time-Dependent Schrodinger Equation

The fundamental equation governing the evolution of a quantum system over time.

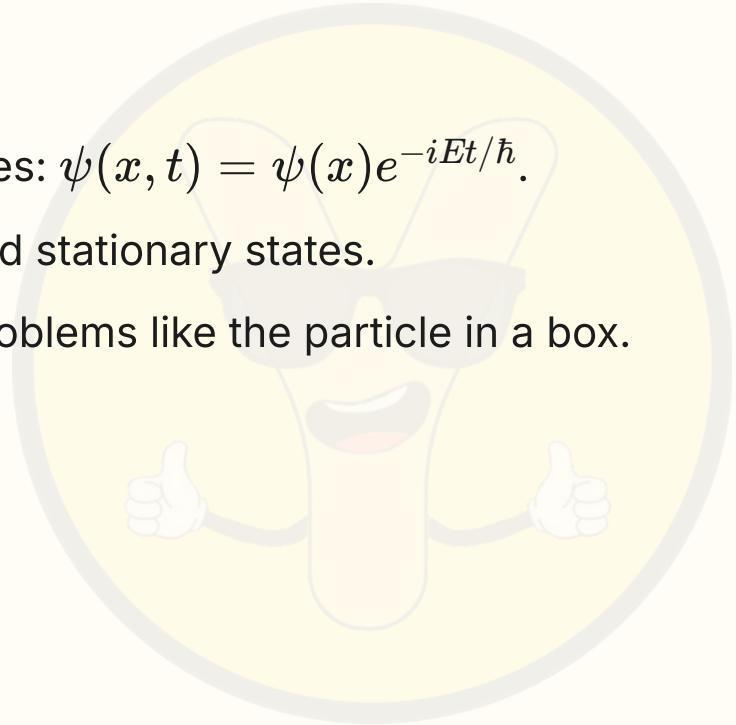
- General form: $i\hbar\frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,t)$.
- $\hat{H}$ is the Hamiltonian operator: $-\frac{\hbar^2}{2m}\nabla^2 + V$.
- It describes how the wave function changes with time.
- Used for time-dependent potentials and non-stationary states.



Time-Independent Schrodinger Equation

Derived for systems where the potential V is independent of time.

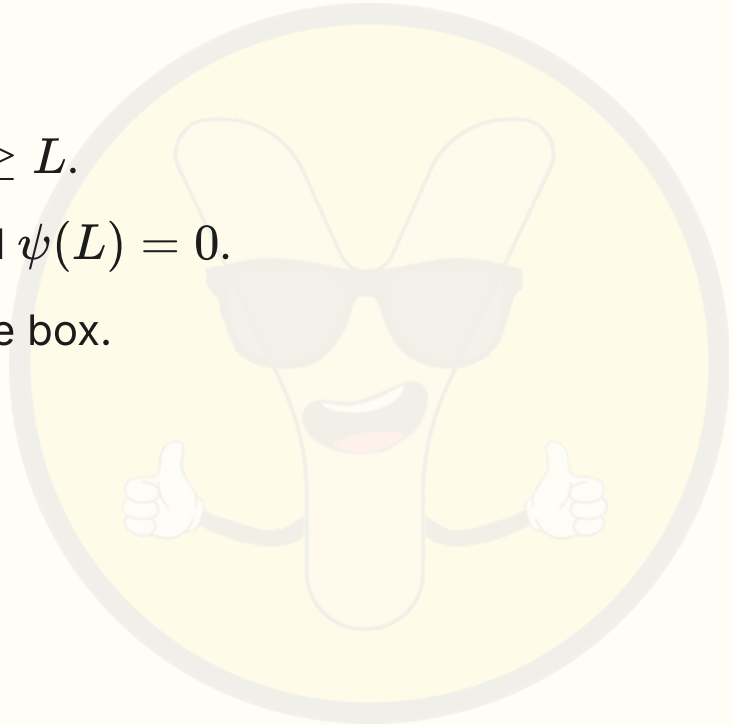
- Form: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$.
- Derived using separation of variables: $\psi(x,t) = \psi(x)e^{-iEt/\hbar}$.
- Used to find energy eigenvalues and stationary states.
- Essential for solving bound state problems like the particle in a box.



Particle in a 1D Box: Setup

A particle of mass m is trapped in a 1D box of length L.

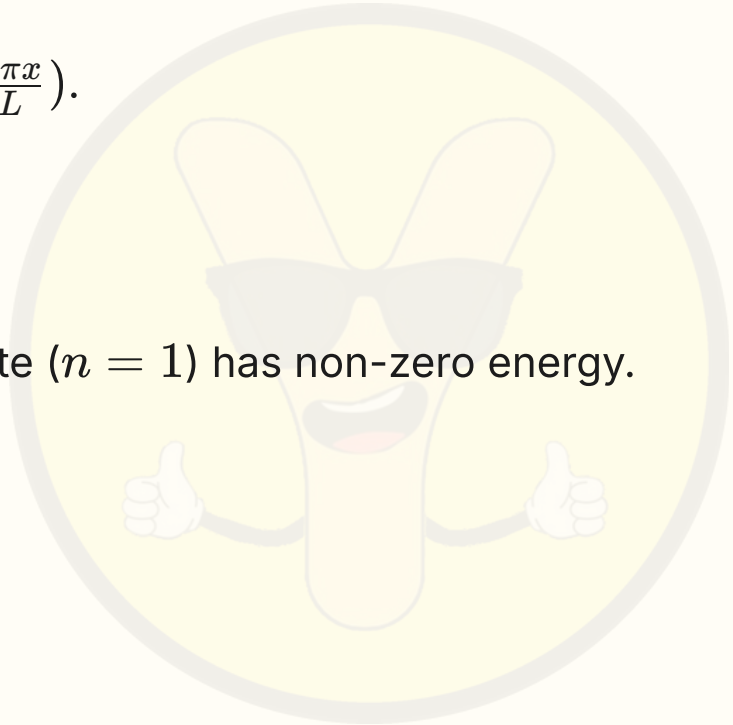
- Potential $V = 0$ for $0 < x < L$.
- Potential $V = \infty$ for $x \leq 0$ and $x \geq L$.
- Boundary conditions: $\psi(0) = 0$ and $\psi(L) = 0$.
- The particle cannot exist outside the box.



Particle in a 1D Box: Solution

Solving the Schrodinger equation yields discrete energy levels.

- Wave function: $\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$.
- Energy eigenvalues: $E_n = \frac{n^2 h^2}{8mL^2}$.
- Quantum number $n = 1, 2, 3, ...$
- Energy is quantized; the ground state ($n = 1$) has non-zero energy.



Exam corner

- **Q1** Derive the de-Broglie wavelength for a particle of mass m and energy E.
- **Q2** Explain the Davisson-Germer experiment and its significance.
- **Q3** Distinguish between phase velocity and group velocity with mathematical proof.
- **Q4** State and explain Heisenberg's uncertainty principle with one application.
- **Q5** Derive the time-independent Schrodinger wave equation.
- **Q6** Solve for the energy eigenvalues of a particle in a 1D infinite potential well.
- **Q7** What are the physical requirements for a valid wave function?
- **Q8** Why is the energy of a particle in a box quantized?

Unit 1 done. You are exam-ready.

Revise the Exam corner once more, then open Unit 2 on theycotes.com. Every AKTU subject, unit by unit, free. Notes by Chirag Jain, Ycotes.

