UNIT–4 : Multipath Mitigation: Equalisation, Diversity and RAKE (PYQ Solution)

PART–A Answers (≈70 Words Each)

Q1. Explain the concept of macrodiversity. (2019)

Macrodiversity is a spatial diversity technique in which the same signal is received (or sent) through several base stations placed far apart (hundreds of metres or more), so that shadowing by buildings or terrain is unlikely to affect all links at once.

The network selects or combines the best signal from the different sites, as in soft handoff in CDMA. It fights large-scale shadowing, whereas microdiversity (antenna spacing of a few wavelengths) fights small-scale fading.

Q2. What is the purpose of an adaptive equalizer? (2025)

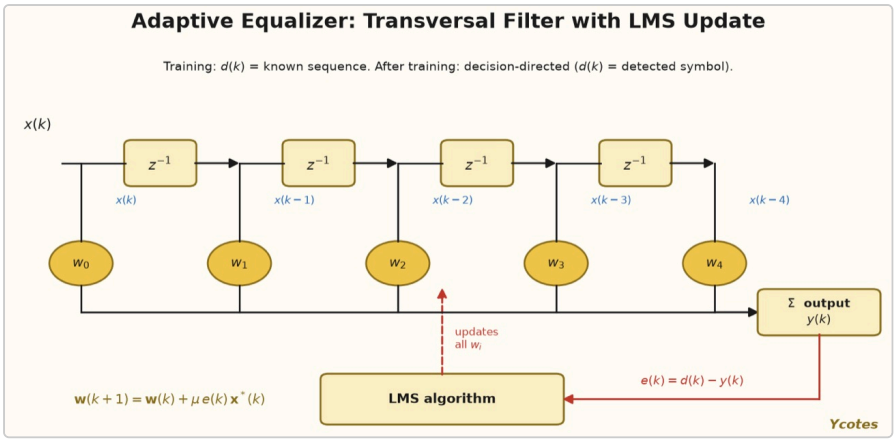
An adaptive equalizer removes inter-symbol interference caused by the time-dispersive (frequency selective) channel. Since the mobile channel changes with time, its tap weights are updated automatically, first with a training sequence and then in decision-directed mode, to track the channel.

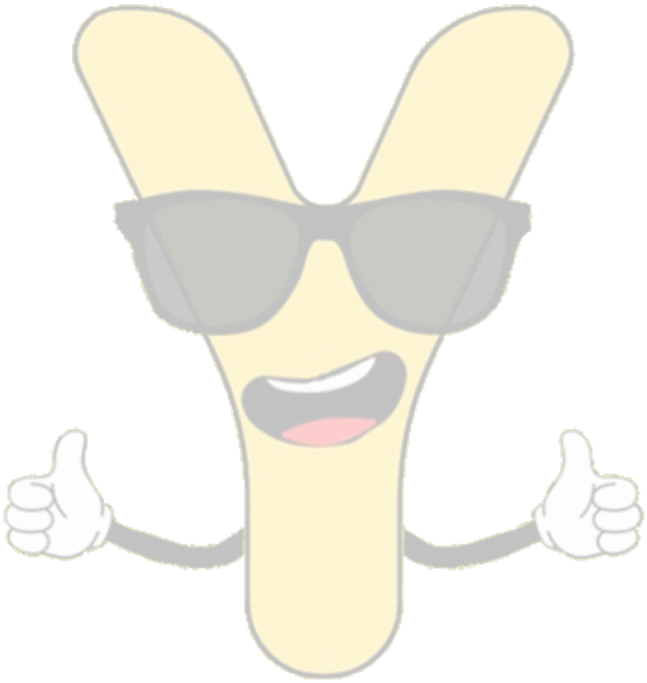
It approximates the inverse of the channel response, improving the BER, and allows a higher data rate without ISI.

Q3. Explain linear equalization. (Feb 2024)

A linear equalizer is a transversal (FIR) filter placed after the receiver filter. Its output is a weighted sum of the present and past received samples, $y(k) = \sum_i w_i \, x(k-i)$, without any feedback from detected symbols.

The weights are chosen by the zero forcing criterion ($W = 1/H$) or by minimum mean square error (MMSE), and adapted by LMS or RLS. It is simple, but it can enhance noise at spectral nulls of the channel, so non-linear equalizers (DFE, MLSE) are used when the channel is severe.

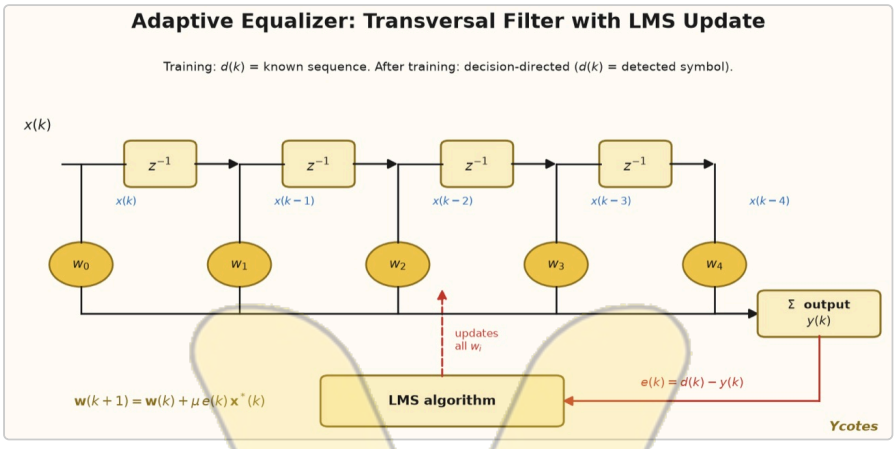

Adaptive Equalizer: Transversal Filter with LMS Update
Training: $d(k)$ = known sequence. After training: decision-directed ($d(k)$ = detected symbol).
$x(k)$
z^{-1}
z^{-1}
z^{-1}
z^{-1}
$x(k)$
$x(k-1)$
$x(k-2)$
$x(k-3)$
$x(k-4)$
w_0
w_1
w_2
w_3
w_4
Σ output
$y(k)$
updates
all w_i
$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\, e(k)\, \mathbf{x}^*(k)$
LMS algorithm
$e(k) = d(k) - y(k)$
Ycotes



PART–B Answers (≈250 Words Each)

Q1. Describe zero forcing and LMS algorithm. (Feb 2024)

Equalizer model: a transversal filter with taps $\mathbf{w} = [w_0, \ldots, w_{N-1}]^T$ and input vector $\mathbf{x}(k)$; output $y(k) = \mathbf{w}^H \mathbf{x}(k)$.



Zero forcing (ZF): the equaliser transfer function is the inverse of the channel, $W(f) = 1/H(f)$, so the overall response is flat and the ISI at the sampling instants is forced to zero. The taps are found by solving $\mathbf{H}\mathbf{w} = \mathbf{e}_0$. It is simple, but where $|H(f)|$ is small the noise is amplified (noise enhancement), so it performs poorly in deep fades.

LMS (least mean square) algorithm: minimises the mean square error $E|e(k)|^2$ by a stochastic gradient descent:

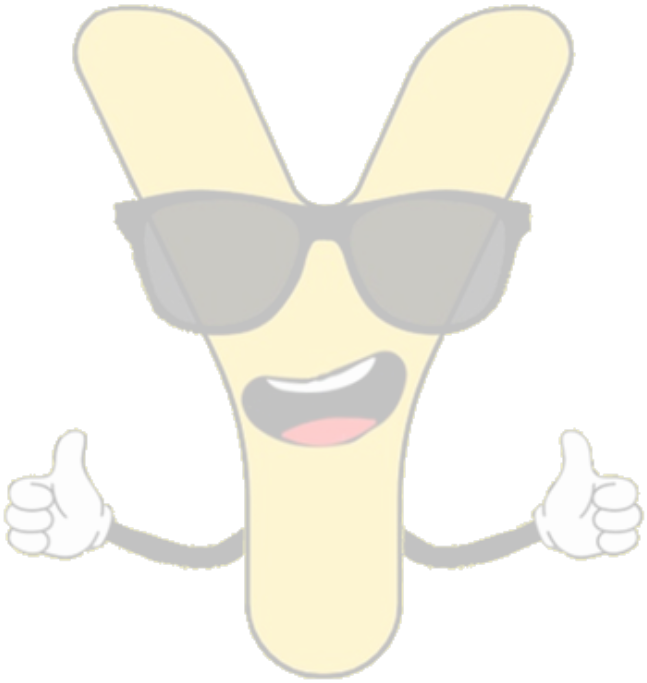
$$e(k) = d(k) - \mathbf{w}^H(k)\mathbf{x}(k)$$

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\, e(k)\, \mathbf{x}^*(k)$$

Here $d(k)$ is the training sequence (later the detected symbols) and μ the step size, with $0 < \mu < 2/\lambda_{max}$. A large μ converges fast but with higher steady-state error; a small μ converges slowly. The complexity is $O(N)$ per iteration.

Example: $\mathbf{w}(0) = [0, 0]$, $\mathbf{x} = [1, 0.5]$, $d = 1$, $\mu = 0.1$: $e = 1$, $\mathbf{w}(1) = [0.1, 0.05]$.

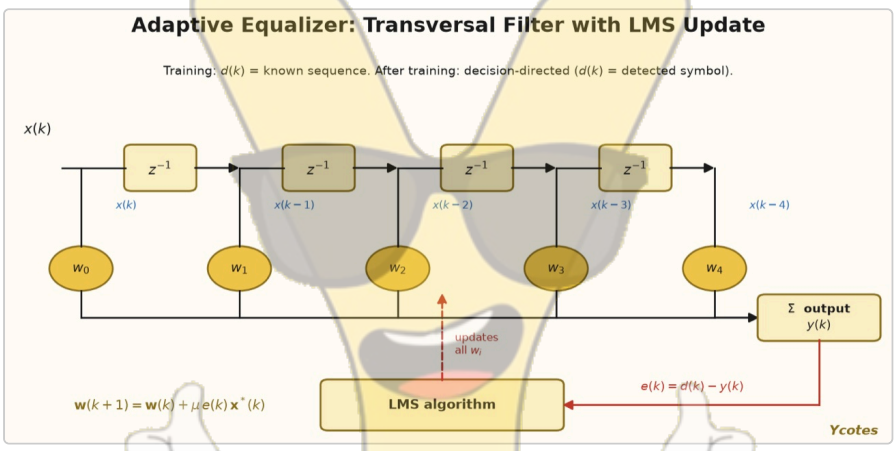
Comparison: ZF is non-adaptive and enhances noise; LMS is adaptive, tracks the channel and does not need the channel inverse.



PART–C Answers (≈500 Words Each)

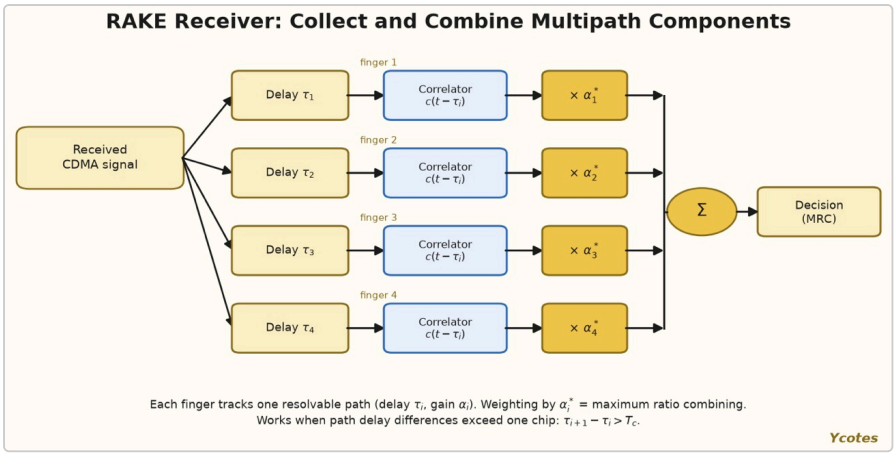
Q1. Explain adaptive equalization, zero forcing and LMS algorithms, and the RAKE receiver. (2019)

Adaptive equalization: mobile channels are time dispersive and time varying, so the equaliser must adapt. A transversal filter followed by a decision device is used, and the weights are updated by an algorithm that minimises the error $e(k) = d(k) - y(k)$. In the **training mode** a known sequence is sent in each burst (e.g. GSM training sequence in the middle of the burst); in the **tracking mode** the detected symbols are used as the desired output (decision-directed). Structures: linear transversal, decision feedback (DFE) and maximum likelihood sequence estimation (MLSE, Viterbi).



Zero forcing: $W(f) = 1/H(f)$; removes ISI completely but amplifies noise. **LMS:** $\mathbf{w}(k+1) = \mathbf{w}(k) + \mu e(k)\mathbf{x}^*(k)$, simple and robust, with the step size μ controlling convergence speed and misadjustment. RLS converges faster but is more complex.

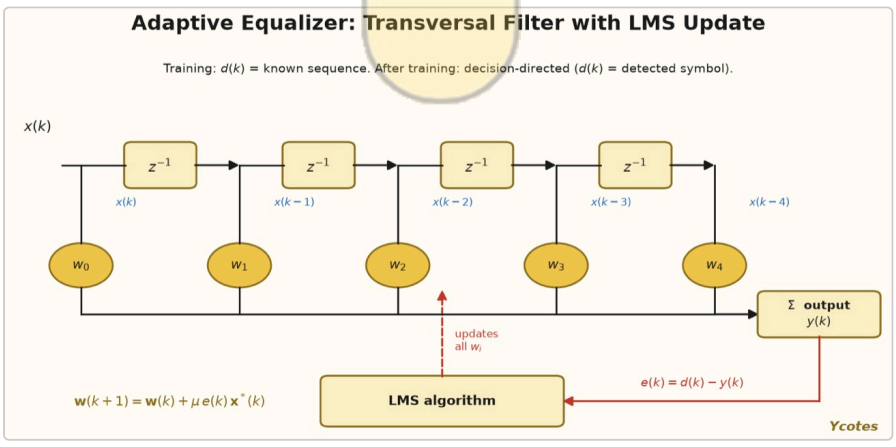
RAKE receiver: in a CDMA system with chip rate larger than the coherence bandwidth, the multipath components are resolvable if their delays differ by more than one chip T_c. The RAKE receiver uses a bank of correlators (**fingers**), each locked to one path with delay τ_i: it despreads using the code delayed by τ_i, multiplies by α_i^* (the conjugate of the path gain), and a combiner sums all fingers (maximum ratio combining). A **searcher** finds the strongest delays.



Result: the receiver gathers the energy of many paths, so multipath acts as **diversity** (order equal to the number of fingers). The output SNR is $\sum_i |\alpha_i|^2 E_b/N_0$. It is used in IS-95 and WCDMA, which also use RAKE for soft handoff.

Q2. Describe the following in detail: Adaptive equalization, Micro-Macro Diversity, Rake receiver. (Feb 2024)

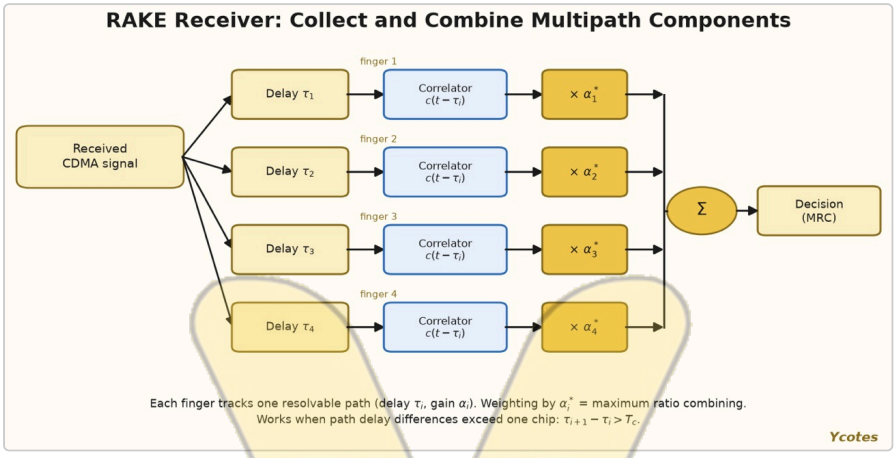
(a) Adaptive equalization: an adaptive equaliser is a filter at the receiver that cancels the ISI introduced by the time-varying dispersive channel. It works in a training phase (known sequence) followed by tracking with decision-directed feedback. The error $e(k) = d(k) - y(k)$ drives the algorithm (LMS, RLS) that updates the tap weights. Types: linear (ZF, MMSE), DFE, and MLSE. In GSM the equaliser handles up to 16 us of delay spread. The cost is complexity and a training overhead.



(b) Micro and macro diversity: Microdiversity counters small-scale fading, using independent copies: spatial (antennas separated by $\geq \lambda/2$), polarisation, frequency (separation greater than B_c), time (separation greater than T_c), and angle diversity. Branches are combined by **selection**, **equal gain** or **maximal ratio combining**,

MRC being optimum with output SNR $\sum \gamma_i$. **Macrodiversity** counters shadowing using base stations at different sites; the best site is selected, or signals are combined, as in soft handoff.

(c) RAKE receiver: a CDMA receiver with several fingers, each tracking a resolvable multipath component using a delayed copy of the spreading code, weighting it with the conjugate channel gain and combining by MRC. It converts the harmful multipath into diversity gain, provided the paths are separated by more than one chip.



Q3. Explain error probability in fading channels with diversity reception. (Feb 2024)

Without diversity: for BPSK in flat Rayleigh fading with mean SNR $\bar{\gamma}$:

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right) \approx \frac{1}{4\bar{\gamma}}$$

The error falls only inversely with SNR, in contrast with exponential decay in AWGN, $Q(\sqrt{2\gamma})$.

With L-branch diversity: with independent Rayleigh branches and maximal ratio combining, the output SNR $\gamma_{MRC} = \sum_{i=1}^{L} \gamma_i$ is chi-square distributed with pdf

$$p(\gamma) = \frac{\gamma^{L-1} e^{-\gamma/\bar{\gamma}}}{(L-1)!\,\bar{\gamma}^{L}}$$

Averaging $Q(\sqrt{2\gamma})$ over this pdf gives

$$P_e = \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k, \quad \mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$$

and at high SNR

$$P_e \approx \binom{2L-1}{L} \left(\frac{1}{4\bar{\gamma}}\right)^L$$

So the error probability decays as $\bar{\gamma}^{-L}$: the **diversity order is** L.

Example at 10 dB per branch: $L = 1$: 2.5×10^{-2}; $L = 2$: 1.9×10^{-3}; $L = 3$: 1.6×10^{-4}; $L = 4$: 1.4×10^{-5}.

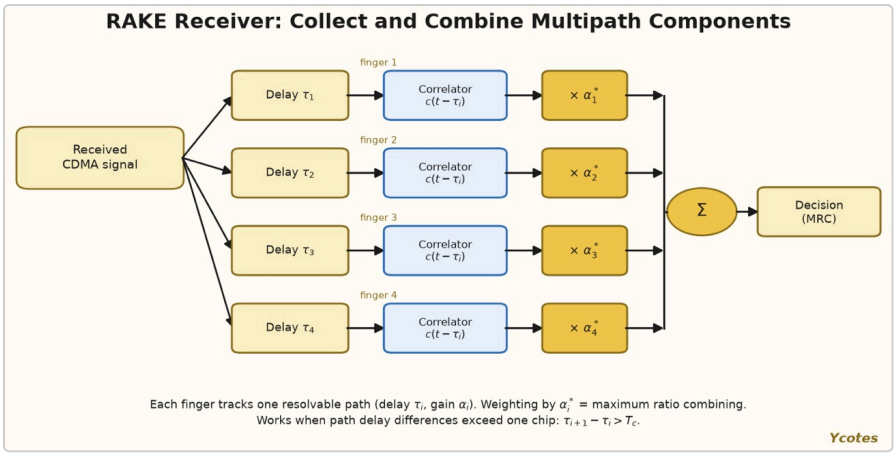
Other combiners: selection diversity has SNR $\bar{\gamma} \sum_{k=1}^{L} 1/k$, equal gain is about 1 dB worse than MRC. Most of the gain comes from the first one or two extra branches.

Q4. Write short notes (any two): (a) Zero forcing scheme (b) RAKE receiver. (2025)

(a) Zero forcing scheme: a linear equalisation method that chooses the equaliser filter as the inverse of the channel, $W(f) = 1/H(f)$, so the cascade of the channel and equaliser has a flat response with zero ISI at the sampling instants. For a transversal filter the taps are obtained by solving the linear equations that make the combined impulse response a unit pulse (1 at the centre tap and 0 at the others).

Advantage: simple, completely removes ISI in a noise-free channel.
Disadvantage: at frequencies where the channel has deep nulls the gain $1/H$ is huge and amplifies the noise, so for a noisy fading channel MMSE equalisation is better. A finite-length filter can only cancel ISI over a limited span.

(b) RAKE receiver: used in CDMA systems to combine resolvable multipath components. It has a set of fingers, each with a correlator using the spreading code delayed by a path delay τ_i estimated by a searcher; the outputs are weighted by α_i^* and added (maximal ratio combining), then a decision is made.



Condition: path separation larger than one chip, $\tau_{i+1} - \tau_i > T_c$.
Benefits: multipath diversity gain with order equal to the number of fingers, improved SNR, helps soft handoff.
Applications: IS-95, WCDMA and UWB receivers.

