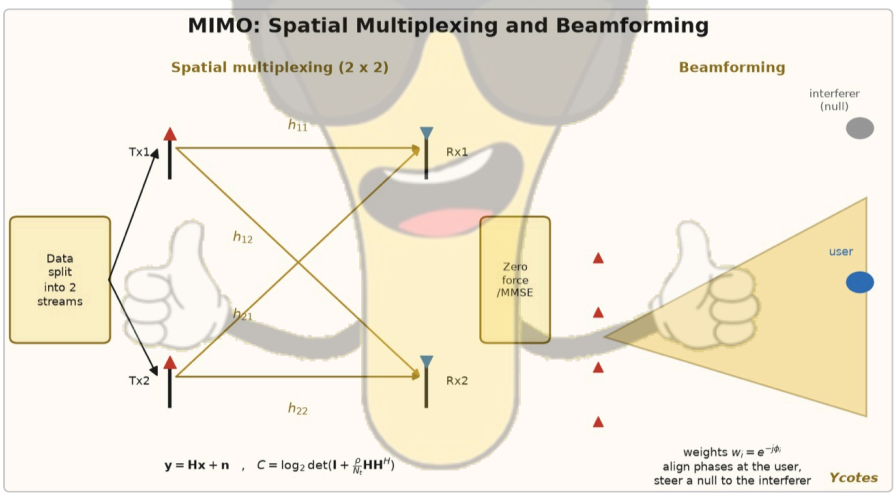
UNIT–5 : Multiple Antennas and MIMO (PYQ Solution)

PART–A Answers (≈70 Words Each)

Q1. What is beam forming in multiple antenna technique? (2019)

Beamforming uses an antenna array whose elements are fed with the same signal multiplied by complex weights w_i (amplitude and phase). The signals add constructively in the direction of the wanted user and cancel in the direction of interferers.

This gives an array gain ($10 \log N$ dB for N elements), a narrow steerable beam, reduced interference and longer range. It needs knowledge of the user direction or channel (CSI).



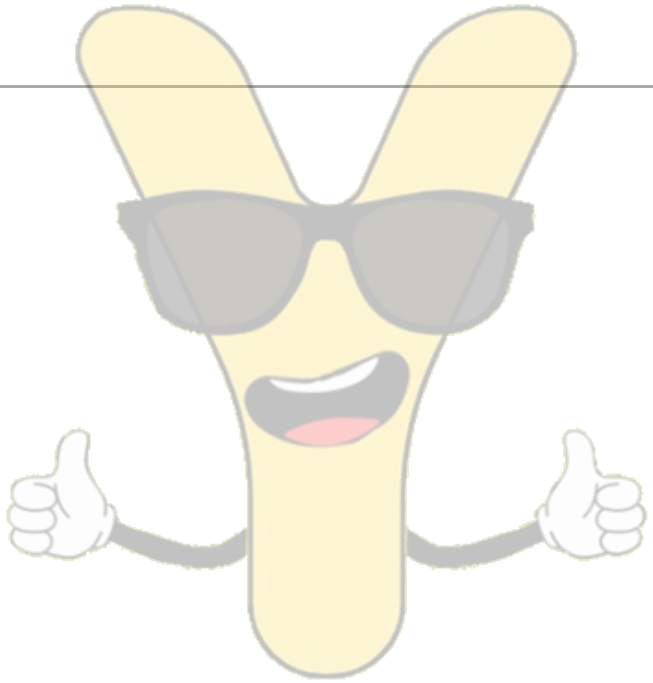
Q2. Define spatial multiplexing in MIMO systems. (2025)

Spatial multiplexing transmits independent data streams from different transmit antennas on the same frequency and at the same time. The scattering environment makes each stream arrive with a different spatial signature, so the receiver, with $N_r \geq N_t$ antennas, separates them with ZF, MMSE or V-BLAST detection.

The data rate (multiplexing gain) increases by $\min(N_t, N_r)$ without extra bandwidth or power.

Q3. What are requirements of a MIMO system? (Feb 2024)

- **Multiple antennas at both the transmitter and receiver ($N_t, N_r > 1$)**
- **A rich scattering environment so the channel gains are uncorrelated**
- **Adequate antenna spacing (about $\lambda/2$ or more) and low correlation**
- **Good SNR, as multiplexing gain needs high SNR**
- **Channel estimation (pilot symbols) at the receiver, and feedback or reciprocity for CSI at the transmitter**
- **Multiple RF chains and a signal processing unit for detection (ZF, MMSE, ML)**



PART–B Answers (≈250 Words Each)

Q1. Describe channel state information (CSI) and capacity in fading and non-fading channels. (2019)

CSI is knowledge of the channel matrix $\mathbf{H}$ (gains and phases). It is estimated at the receiver (**CSIR**) with pilot symbols and may be fed back (FDD) or inferred by reciprocity (TDD) to give **CSIT**. With CSIT the transmitter can use beamforming, precoding and water-filling; without it, it uses space-time codes.

Non-fading (AWGN) channel:

$$C = B\log_2(1+\mathrm{SNR})\ \mathrm{bit/s}$$

with a fixed SNR; error-free communication is possible at any rate below C.

Fading channel (SISO): with gain h the instantaneous capacity is $B\log_2(1+|h|^2\mathrm{SNR})$.

- **Fast fading (ergodic capacity):** $C = B\,E_h[\log_2(1+|h|^2\mathrm{SNR})]$. By Jensen's inequality this is less than the AWGN capacity at the same average SNR.
- **Slow fading:** capacity is random, so the **outage probability** $P_{out} = P(\log_2(1+|h|^2\mathrm{SNR}) < R)$ is used.
- With CSIT the transmitter can adapt power (water-filling) and rate to gain further.

MIMO capacity:

$$C = \log_2\det\left(\mathbf{I} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right)\ \mathrm{bit/s/Hz}\ \ (\text{no CSIT})$$

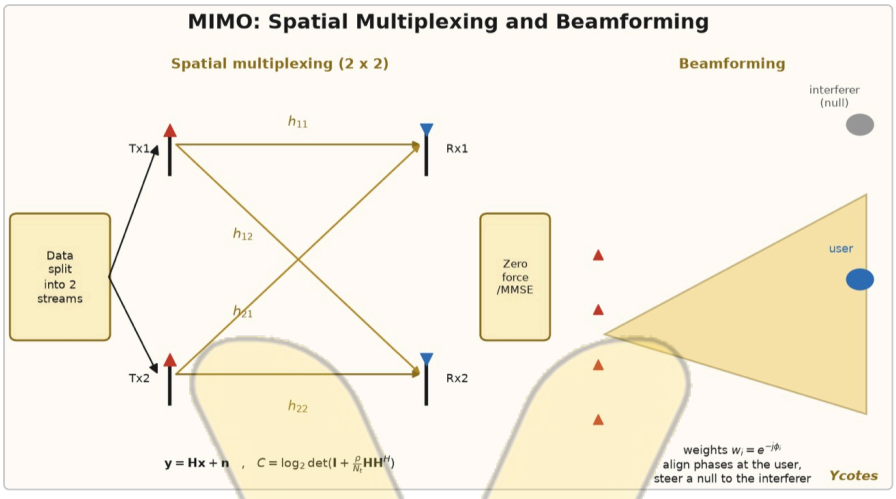
Capacity grows linearly with $\min(N_t, N_r)$ in a rich scattering channel. Example at $\rho = 100$: SISO $\log_2 101 = 6.66$; 2x2 with independent paths $2\log_2(1+50) = 11.34$ bit/s/Hz.

Q2. Explain beam forming in multiple antenna technique. (Feb 2024)

Concept: an array of N elements spaced about $\lambda/2$ apart transmits the same symbol s with element weights $w_i = a_i e^{j\phi_i}$. The array factor

$$AF(\theta) = \sum_{i=0}^{N-1} w_i \, e^{jkid\cos\theta}$$

is made to peak in the desired direction θ_0 by choosing the phase $\phi_i = -kid\cos\theta_0$. Waves from all elements then arrive in phase at the user.



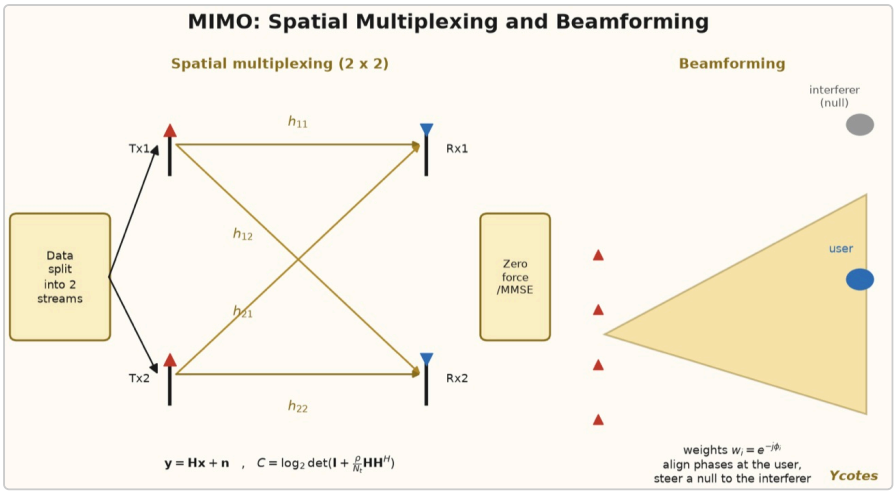
Spatial nulls: weights can be chosen to place nulls at the interferers (adaptive or null-steering beamforming).

With channel knowledge: for a MISO channel $\mathbf{h}$, the maximum ratio transmission weight vector is $\mathbf{w} = \mathbf{h}^*/\|\mathbf{h}\|$, giving SNR $\|\mathbf{h}\|^2 P/N_0$.

Advantages: array gain $10\log N$ dB, larger range, interference suppression, higher capacity (SDMA), lower power. **Limitations:** needs accurate direction or CSI, and beam width narrows as N grows, so tracking mobiles is harder.

Q3. Explain Spatial Multiplexing in detail? (Feb 2024)

Idea: use the extra spatial dimension of a rich scattering MIMO channel to send N_t independent streams simultaneously on the same frequency, increasing the data rate by up to $\min(N_t, N_r)$.



System model: $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$, where $\mathbf{H}$ is $N_r \times N_t$. For a 2x2 system

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Detection:

- **Zero forcing:** $\hat{\mathbf{x}} = \mathbf{H}^{+}\mathbf{y} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{y}$, simple but enhances noise if $\mathbf{H}$ is ill conditioned
- **MMSE:** $\hat{\mathbf{x}} = (\mathbf{H}^H\mathbf{H} + \sigma^2\mathbf{I})^{-1}\mathbf{H}^H\mathbf{y}$
- **V-BLAST (successive interference cancellation):** detect the strongest stream, subtract, repeat
- **ML:** best but complexity grows exponentially

Capacity: $C = \log_2 \det(\mathbf{I} + \frac{\rho}{N_t}\mathbf{HH}^H) \approx \min(N_t, N_r)\log_2 \rho$ at high SNR.

Trade-off: diversity and multiplexing gain cannot both be maximised; spatial multiplexing needs high SNR and uncorrelated paths, while diversity coding (Alamouti) is better at low SNR.

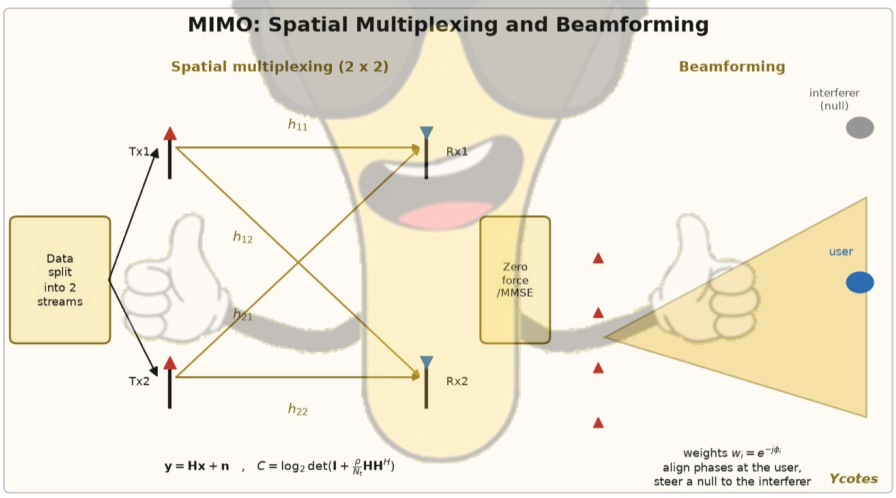
PART–C Answers (≈500 Words Each)

Q1. Describe the following in detail: (a) MIMO systems (b) Spatial multiplexing (c) Transmitter and receiver diversity. (2019)

(a) MIMO systems: a link with N_t transmit and N_r receive antennas, modelled as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where h_{ij} is the complex gain from transmit antenna j to receive antenna i (Rayleigh for rich scattering). MIMO gives three benefits: **array gain** (beamforming), **diversity gain** (up to $N_t N_r$) and **multiplexing gain** (up to $\min(N_t, N_r)$). Capacity:

$$C = \log_2 \det\left(\mathbf{I} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right)$$

In 4G and 5G, MIMO is a key technology (up to 8 x 8 and massive MIMO with 64 or more antennas).



(b) Spatial multiplexing: the data stream is split into N_t sub-streams sent simultaneously from different antennas in the same band. The receiver recovers them from $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$ using ZF $\mathbf{H}^{+}\mathbf{y}$, MMSE or V-BLAST (successive cancellation), requiring $N_r \geq N_t$ and a good, well-conditioned $\mathbf{H}$. The data rate is multiplied by $\min(N_t, N_r)$ with no extra bandwidth or power. With CSIT, SVD precoding transmits along eigenmodes with water-filling power allocation.

(c) Transmitter and receiver diversity:

- **Receive diversity (SIMO, $1 \times N_r$):** the N_r antenna outputs are combined by MRC, $z = \sum h_i^* r_i$, giving array gain N_r and diversity order N_r. Error

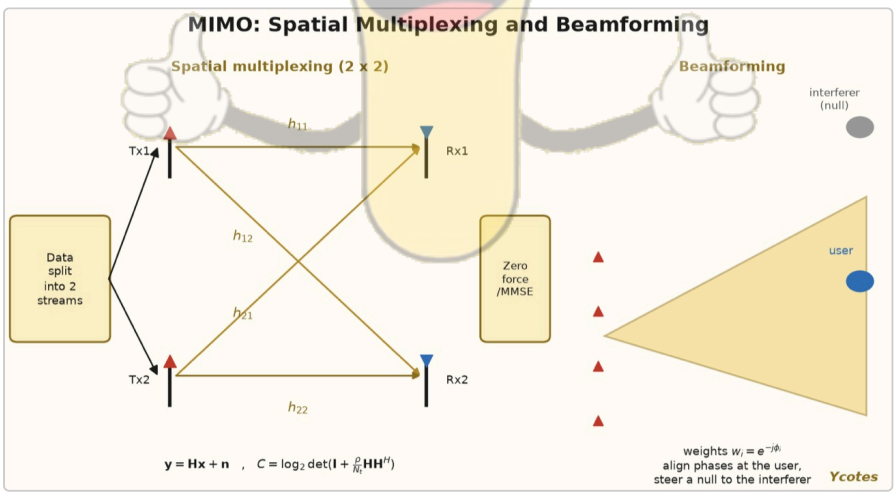
probability $\propto \bar{\gamma}^{-N_r}$.

- **Transmit diversity (MISO, $N_t \times 1$):** transmit the same information through several antennas using space-time coding without CSIT. In the **Alamouti scheme (2x1)**, over two symbol times antenna 1 sends $s_1,\ -s_2^*$ and antenna 2 sends $s_2,\ s_1^*$. The receiver combines the two received samples to obtain $(|h_1|^2 + |h_2|^2)s_i$ plus noise, so diversity order 2 is achieved at full rate with simple linear processing.
- With both ends, MIMO diversity is $N_t N_r$.

Trade-off: diversity improves reliability, multiplexing improves rate; a system chooses between them depending on SNR and channel conditions.

Q2. Explain the concept of multi-antenna systems and compute the optimum weighting coefficients for multi antenna system employing maximum ratio combining scheme. (2025)

Multi-antenna systems: using several antennas at the transmitter, the receiver or both, exploits the spatial domain to improve reliability (diversity gain), coverage (array gain), interference suppression (beamforming) and data rate (spatial multiplexing). Configurations: SISO, SIMO (receive diversity), MISO (transmit diversity) and MIMO.



Maximum ratio combining (MRC): with L receive branches, the received signal on branch i is

$$r_i = h_i s + n_i, \quad i = 1, \ldots, L$$

where $h_i = \alpha_i e^{j\theta_i}$ is the channel gain and n_i is noise of variance N_0. The combiner output is

$$z = \sum_{i=1}^{L} w_i^* r_i = s \sum_i w_i^* h_i + \sum_i w_i^* n_i$$

Output SNR:

$$\gamma = \frac{E_s \left| \sum w_i^* h_i \right|^2}{N_0 \sum |w_i|^2}$$

Optimum weights: by the Cauchy-Schwarz inequality $\left| \sum w_i^* h_i \right|^2 \le \sum |w_i|^2 \sum |h_i|^2$, with equality when $\mathbf{w} = c\,\mathbf{h}$ (proportional to the channel). Hence the optimum weights are

$$w_i = \frac{h_i}{\sqrt{\sum_j |h_j|^2}} \quad (\text{so } w_i^* = \alpha_i e^{-j\theta_i} \text{ up to a constant})$$

i.e. each branch is **co-phased** (phase $-\theta_i$) and **weighted by its amplitude** α_i: stronger branches get larger weights.

Maximum SNR:

$$\gamma_{MRC} = \frac{E_s}{N_0} \sum_{i=1}^{L} |h_i|^2 = \sum_i \gamma_i$$

so the output SNR is the sum of the branch SNRs; with equal mean SNR it gives an array gain of L.

Example: $h = (0.8,\ 0.6)$: $w = (0.8, 0.6)$ and $\gamma = (0.64 + 0.36) E_s/N_0 = 1.0\, E_s/N_0$. The best single branch gives 0.64, and equal gain combining $(0.8 + 0.6)^2/2 = 0.98$. MRC is therefore optimum.
